

**CURRICULUM FOR MATHEMATICS
IN UNDER GRADUATE DEGREE PROGRAMME**

**CBCS SYLLABUS SCHEDULE 2016 - 2017
SEMESTER - V**



**By
Chairperson
Board of Studies
Department of Mathematics
Kakatiya University, Warangal.**

Skill Enhancement Course - III
B.Sc., III Year, V Semester
For All Science Faculty Departments
Verbal Reasoning For Aptitude Test
Credits: 2

Theory: 2 hours/week

Marks - 50

UNIT - I - Numbers And Diagrams

- 1.1. **Series Completion:** Number series, Alphabet Series.
- 1.2. **Series Completion:** Alpha Numeric Series, Continuous Pattern Series.
- 1.3. **Logical Venn Diagrams.**
- 1.4. **Mathematical Operations:** Problem solving by substitution, Interchange of signs and numbers.

UNIT - II - Arithmetical Reasoning

- 2.1. **Mathematical Operations:** Deriving the appropriate conclusions.
- 2.2. **Arithmetical Reasoning:** Calculation based problems, Data based problems .
- 2.3. **Arithmetical Reasoning:** Problems on ages, Venn diagram based problems.
- 2.4. **Cause and Effect Reasoning.**

TEXT: *A Modern Approach to Verbal and Non-Verbal Reasoning* by Dr.R.S. Aggarwal

Kakatiya University
B.Sc. Mathematics, V Semester
LINEAR ALGEBRA

DSC-1E
BS:503

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: The students are exposed to various concepts like vector spaces, bases, dimension, Eigen values etc.

Outcome: After completion this course students appreciate its interdisciplinary nature.

UNIT-I

Vector Spaces : Vector Spaces and Subspaces -Null Spaces, Column Spaces, and Linear Transformations -Linearly Independent Sets; Bases -Coordinate Systems

UNIT-II

The Dimension of a Vector Space, Rank-Change of Basis - Eigenvalues and Eigenvectors .

UNIT-III

The Characteristic Equation, Diagonalization -Eigenvectors and Linear Transformations -Complex Eigenvalues - Applications to Differential Equations .

UNIT-IV

Orthogonality and Least Squares : Inner Product, Length, and Orthogonality -Orthogonal Sets.

TEXT: David C Lay,*Linear Algebra and its Applications 4e*

References:

- S Lang, *Introduction to Linear Algebra*
- Gilbert Strang ,*Linear Algebra and its Applications*
- Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence; *Linear Algebra*
- Kuldeep Singh; *Linear Algebra*
- Sheldon Axler;*Linear Algebra Done Right*

Practical Question Bank

UNIT-I

- (1) Let H be the set of all vectors of the form $\begin{bmatrix} -2t \\ 5t \\ 3t \end{bmatrix}$. Find a vector v in R^3 such that $H = \text{Span}\{v\}$.
Why does this show that H is a subspace of R^3 ?
- (2) Let V be the first quadrant in the xy -plane; that is let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x \geq 0, y \geq 0 \right\}$
- (a). If u and v are in V is $u + v$ in V ? why?
(b) Find a specific vector u in V and a specific scalar c such that
- (3) Let $v_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -2 \\ 7 \\ -9 \end{bmatrix}$. Determine if $\{v_1, v_2\}$ is a basis for R^3 . Is $\{v_1, v_2\}$ a basis for R^2 .
- (4) The set $B = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for P_2 . Find the coordinate vector of $p(t) = 1 + 4t + 7t^2$ relative to B .
- (5) set $B = \{1 - t^2, t - t^2, 2 - t + t^2\}$ is a basis for P_2 . Find the coordinate vector of $p(t) = 1 + 3t - 6t^2$ relative to B .
- (6) The vector $v_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ -8 \end{bmatrix}, v_3 = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ span R^2 but do not form a basis. Find two different ways to express $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as a linear combination of v_1, v_2, v_3
- (7) Let V be the set of all real-valued functions defined on a set D . Then $f + g$ is the function whose value at t in the domain D is $f(t) + g(t)$ and for any scalar c and for any f in V , the scalar multiple cf is the function whose value at t is $cf(t)$.
- (8) The vector space \mathbf{R}^2 is not a subspace of \mathbf{R}^3 because \mathbf{R}^2 is not even a subset of \mathbf{R}^3 . (The vectors in \mathbf{R}^3 all have three entries, whereas the vectors in \mathbf{R}^2 have only two.) The set $\mathbf{H} = \left\{ \begin{pmatrix} s \\ t \\ 0 \end{pmatrix} : s \text{ and } t \text{ are real} \right\}$ is a subset of \mathbf{R}^3 that "looks" and "acts" like \mathbf{R}^2 , although it is logically distinct from \mathbf{R}^2 . Show that H is a subspace of \mathbf{R}^3 .
- (9) The differential equation $y'' + \omega^2 y = 0$ where ω is a constant, is used to describe a variety of physical systems, such as the vibration of a weighted spring, the movement of a pendulum, and the voltage in an inductance-capacitance electrical circuit. Then show that the set of solutions of the given differential equation is precisely the kernel of the linear transformation that maps a function $y = f(t)$ into the function $y'' + \omega^2 y = 0$.
- (10) Let $v_1 = \begin{pmatrix} 3 \\ 0 \\ -6 \end{pmatrix}, v_2 = \begin{pmatrix} -4 \\ 1 \\ 7 \end{pmatrix}, v_3 = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix}$. Determine if $\{v_1, v_2, v_3\}$ is a basis for \mathbf{R}^3

UNIT-II

- (11) Find the dimension of the subspace of all vectors in R^3 whose first and third entries are equal
- (12) Find the dimension of the subspace H of R^2 spanned by $\begin{bmatrix} 1 \\ -5 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 10 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 15 \end{bmatrix}$
- (13) Let H be an n dimensional subspace of an n dimensional vectorspace V . Show that $H=V$.
- (14) Explain why the space P of all polynomials is an infinite dimensional space
- (15) If a 4×7 matrix A has rank 3 ,find $\dim \text{Null}A$, $\dim \text{Row } A$ and rank A^T
- (16) If a 7×5 matrix A has rank 2 ,find $\dim \text{Null}A$, $\dim \text{Row } A$ and rank A^T
- (17) If the null space of an 8×5 matrix A is 3 dimensional,what is the dimension of the row space of A ?
- (18) If A is a 3×7 matrix what is the smallest possible dimension of $\text{Null } A$?
- (19) Let $U = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ find V in R^3 such that $\begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \end{bmatrix} = UV^T$
- (20) If A is a 7×5 matrix,what is the largest possible rank of A ? If A is a 5×7 matrix,what is the largest possible rank of A ? Explain your answers.

UNIT-III

- (21) Without calculations list $\text{rank}(A)$ and $\dim(A)$, $\text{Nul}(A)$

$$\text{if } A = \begin{bmatrix} 2 & 6 & -6 & -6 & 3 & 6 \\ -2 & -3 & 6 & -3 & 0 & -6 \\ 4 & 9 & 12 & 9 & 3 & 12 \\ -2 & 3 & 6 & 3 & 3 & -6 \end{bmatrix}$$

- (22) Use a property of determinants to show A and A^T have same characteristic polynomial.
- (23) Find the characteristic equation of

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- (24) Find characteristic polynomial and the real eigen values of

$$\begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

- (25) Let $A = PDP^{-1}$ and compute A^4 where $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$
- (26) Let $B = \{b_1, b_2, b_3\}$ and $D = \{d_1, d_2\}$ be bases for vector spaces V and W respectively. Let $T : V \rightarrow W$ be a linear transformation with the property that $T(b_1) = 3d_1 - 5d_2$, $T(b_2) = -d_1 + 6d_2$, $T(b_3) = 4d_2$ Find the matrix T relative to B and D .

(27) Let $D = \{d_1, d_2\}$ and $B = \{b_1, b_2\}$ be bases for vector spaces V and W respectively. Let $T : V \rightarrow W$ be a linear transformation with the property that $T(d_1) = 3b_1 - 3b_2$, $T(d_2) = -2b_1 + 5b_2$. Find the matrix for T relative to B and D .

(28) Let $B = \{b_1, b_2, b_3\}$ be a basis for a vector space V and let $T : V \rightarrow \mathbf{R}^2$ be a linear transformation with the property that

$$T(x_1b_1 + x_2b_2 + x_3b_3) = \begin{bmatrix} 2x_1 - 3x_2 + x_3 \\ -2x_1 + 5x_3 \end{bmatrix}$$

find the matrix for T relative to B and the standard basis for \mathbf{R}^2 .

(29) Let $T : P_2 \rightarrow P_3$ be the transformation that maps a polynomial $p(t)$ into the polynomial $(t+3)p(t)$

(a). Find the image of $p(t) = 3 - 2t + t^2$

(b). Show that T is a linear transformation

(c). Find the matrix for T relative to the basis $\{1, t, t^2\}$ and $\{1, t, t^2, t^3\}$

(30) Assume the mapping $T : P_2 \rightarrow P_2$ defined by

$T(a_0 + a_1t + a_2t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$ is linear. Find the matrix representation of T relative to the basis $B = \{1, t, t^2\}$

UNIT-IV

(31) Define $T : P_3 \rightarrow \mathbf{R}^4$ by $T(P) = \begin{bmatrix} P(-2) \\ P(3) \\ P(1) \\ P(0) \end{bmatrix}$

(a. Show that T is a linear transformation

(b. Find the matrix for T relative to the basis $\{1, t, t^2, t^3\}$ for P_3 and standard basis for \mathbf{R}^4

(32) Let A be 2×2 matrix with eigen values -3 and -1 corresponding eigen vectors $V_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

and $V_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Let $X(t)$ be the position of a particle at time t solve the initial value problem

$$X' = AX, X(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

(33) Construct the general solution of $X' = AX, A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -7 & 10 \\ -4 & 5 \end{bmatrix}$

(34) Compute the orthogonal projection of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ onto the line through $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and the origin.

(35) Let W be the subspace of \mathbf{R}^2 spanned by $X = (\frac{2}{3}, 1)$. Find a unit vector in z that is a basis for W .

(36) Show that $\{u_1, u_2, u_3\}$ is an orthogonal set, where $u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} -\frac{1}{2} \\ -2 \\ \frac{7}{2} \end{pmatrix}$.

(37) The set $S = \{u_1, u_2, u_3\}$ where $u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$, $u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $u_3 = \begin{pmatrix} -\frac{1}{2} \\ -2 \\ \frac{7}{2} \end{pmatrix}$ is an orthogonal basis for R^3 . Express the vector $y = \begin{pmatrix} 6 \\ 1 \\ -8 \end{pmatrix}$ as a linear combination of the vectors in S .

(38) Show that $S = \{v_1, v_2, v_3\}$ is an orthonormal basis of R , where $v_1 = \begin{pmatrix} \frac{3}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \end{pmatrix}$, $v_2 = \begin{pmatrix} -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$,
 $v_3 = \begin{pmatrix} -\frac{1}{\sqrt{66}} \\ \frac{4}{\sqrt{66}} \\ \frac{7}{\sqrt{66}} \end{pmatrix}$

(39) Determine given set of vectors are orthogonal or not. $\begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$, $\begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -4 \\ -7 \end{pmatrix}$

(40) Let $U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{3} \\ \frac{1}{\sqrt{2}} & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$ and $x = \begin{bmatrix} \sqrt{2} \\ 3 \end{bmatrix}$. Notice that U has orthonormal columns and $U^T U =$
 $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{3} \\ \frac{1}{\sqrt{2}} & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ verify that $\|Ux\| = \|x\|$.

Kakatiya University
B.Sc. Mathematics, V Semester
SOLID GEOMETRY

DSE-1E/A
BS:506

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours /week and Practicals: 2 hours/week

Objective: Students learn to describe some of the surfaces by using analytical geometry.

Outcome: Students understand the beautiful interplay between algebra and geometry.

UNIT- I

Sphere: Definition-The Sphere Through Four Given Points - Equations of a Circle - Intersection of a Sphere and a Line - Equation of a Tangent Plane - Angle of Intersection of Two Spheres - Radical Plane.

UNIT- II

Cones : Definition-Condition that the General Equation of second degree Represents a Cone - Cone and a Plane through its Vertex - Intersection of a Line with a Cone. The Right Circular Cone.

UNIT- III

Cylinder: Definition-Equation of a Cylinder-Enveloping Cylinder - The Cylinder - The Right Circular Cylinder.

UNIT- IV

The Conicoid: The General Equation of the Second Degree-Intersection of Line with a Conicoid- Plane of contact-Enveloping Cone and Cylinder.

TEXT: Shanti Narayan and P K Mittal, *Analytical Solid Geometry* (17e)

References:

- Khaleel Ahmed, *Analytical Solid Geometry*
- S L Loney , *Solid Geometry*
- Smith and Minton, *Calculus*

Practical Question Bank

UNIT-I

- (1) Find the equation of the sphere through the four points $(4,-1,2)$, $(0,-2,3)$, $(1,-5,-1)$, $(2,0,1)$.
- (2) Find the equation of the sphere through the four points $(0,0,0)$, $(-a,b,c)$, $(a,-b,c)$, $(a,b,-c)$.
- (3) Find the centre and radius of the circle $x + 2y + 2 = 15$, $x^2 + y^2 + z^2 - 2y - 4z = 11$.
- (4) Show that the following points are concyclic:
 - (i) $(5,0,2)$, $(2,-6,0)$, $(7,-3,8)$, $(4,-9,6)$.
 - (ii) $(-8,5,2)$, $(-5,2,2)$, $(-7,6,6)$, $(-4,3,6)$.
- (5) Find the centres of the two spheres which touch the plane $4x + 3y = 47$ at the points $(8.5,4)$ and which touch the sphere $x^2 + y^2 + z^2 = 1$
- (6) Show that the spheres $x^2 + y^2 + z^2 = 25$ & $x^2 + y^2 + z^2 - 24x - 40y - 18z + 225 = 0$ touch externally and find the point of contact.
- (7) Find the equation of the sphere that passes through the two points $(0,3,0)$, $(-2,-1,-4)$ and cuts orthogonally the two spheres
 $x^2 + y^2 + z^2 - x - 3z - 2 = 0$, $2(x^2 + y^2 + z^2) + x + 3y + 4 = 0$.
- (8) Find the limiting points of the co-axial system of spheres $x^2 + y^2 + z^2 - 20x + 30y - 40z + 29 + \lambda(2x - 3y + 4z) = 0$.
- (9) Find the equation of the two spheres of the co-axial systems $x^2 + y^2 + z^2 - 5 + \lambda(2x + y + 3z - 3) = 0$ which touch the plane $3x + 4y = 15$.
- (10) Show that the radical planes of the spheres of a co-axial system and of any given sphere pass through a line.

UNIT-II

- (11) Find the equation of cone whose vertex is (α, β, γ) and base $ax^2 + by^2 = 1$, $z = 0$
- (12) The section of a cone whose vertex is P and guiding curve the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ by the plane $x = 0$ is a rectangular hyperbola . show that the locus of P is $\frac{x^2}{a^2} + \frac{y^2+z^2}{b^2} = 1$
- (13) Find the equation of the cone whose vertex is the point $(1,1,0)$ and whose guiding curve is $y = 0, x^2 + y^2 = 4$
- (14) Find the equation of the cone whose vertex is the point $(1,2,3)$ and guiding curve the circle $x^2 + y^2 + z^2 = 4, x + y + z = 1$
- (15) Find the enveloping cone of the sphere $x^2 + y^2 + z^2 - 2x + 4z = 1$ with vertex at $(1,1,1)$.
- (16) Show that the plane $z = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has its vertex at $(2,4,1)$ in a rectangular hyperbola.
- (17) Find the equation of the quadric cone whose vertex is at the origin and which passes through the curve given by the equations $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = p$.

- (18) Find the equations to the cones with vertex at origin and which pass through the curve given by the equations $ax^2 + by^2 = 2z, lx + my + nz = p$.
- (19) Find the equation of the cone with vertex at the origin and direction cosines of its generators satisfying the relation $3l^2 - 4m^2 + 5n^2 = 0$
- (20) Find the equations to the cones with vertex at origin and which pass through the curve given by the equations $z = 2, x^2 + y^2 = 4$

UNIT-III

- (21) Find the equation of a cylinder whose generating line have the direction cosines (l, m, n) and which passes through the circle $x^2 + z^2 = a^2, y = 0$.
- (22) Find the equation of the cylinder whose generators are parallel to $x = -\frac{1}{2}y = \frac{1}{3}z$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$.
- (23) Find the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y = 1$ having the generators parallel to the line $x = y = z$.
- (24) The axis equation of a right circular cylinder of radius 2 is $\frac{(x-1)}{2} = \frac{y}{3} = \frac{(z-3)}{1}$;
Show that its equation is $10x^2 + 5y^2 + 13z^2 - 12xy - 6yz - 4zx - 8x - 30y - 74z + 59 = 0$.
- (25) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-2}{2}$.
- (26) Find the equation of the right circular cylinder of radius 2 whose axis passes through the point $(1, 2, 3)$ and has direction cosines proportional to $(2, -3, 6)$.
- (27) Find the right circular cylinder of radius 4 and axis the line $x = 2y = -z$. Also prove that the area of cross - section of the cylinder by the plane $z = 0$ is 24π
- (28) Obtain the equation of the right circular cylinder described on the circle through the three points $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ as guiding circle.
- (29) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{(x-1)}{2} = (y-2) = \frac{(z-3)}{2}$
- (30) Find the equation, if the cylinder whose generator touch the sphere $x^2 + y^2 + z^2 = a^2$ and parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

UNIT-IV

- (31) Find the points of intersection of the line $-\frac{1}{3}(x + 5) = (y - 4) = \frac{1}{7}(z - 11)$, with the conicoid $12x^2 - 17y^2 + 7z^2 = 7$.
- (32) Find the equations to the tangent planes to $7x^2 - 3y^2 - z^2 + 21 = 0$, which passes through the line, $7x - 6y + 9 = 3, z = 3$.
- (33) Obtain the tangent planes to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, which are parallel to the plane $lx + my + nz = 0$.
- (34) Show that the plane $3x + 12y - 6z - 17 = 0$ touches the conicoid $3x^2 - 6y^2 + 9z^2 + 17 = 0$, and find point of contact.

- (35) Find the equations to the tangent planes to the surface $4x^2 - 5y^2 + 7z^2 + 13 = 0$ parallel to the plane $4x + 20y - 21z = 0$. Find their points of contact also.
- (36) Find the locus of the perpendiculars from the origin to the tangent planes to the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which cut off from its axes intercepts the sum of whose reciprocals is equal to a constant $\frac{1}{k}$.
- (37) If the section of the enveloping one of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, whose vertex is P by the plane $z = 0$ is a rectangular hyperbola, show that the locus of P is $\frac{x^2+y^2}{a^2+b^2} + \frac{z^2}{c^2} = 1$.
- (38) Find the locus of points from which three mutually perpendicular tangent lines can be drawn to the conicoid $ax^2 + by^2 + cz^2 = 1$.
- (39) $P(1, 3, 2)$ is a point on the conicoid $x^2 - 2y^2 + 3z^2 + 5 = 0$. Find the locus of the mid-points of chords drawn parallel to OP .

Kakatiya University
B.Sc. Mathematics, V Semester
INTEGRAL CALCULUS

DSE-1E/B
BS:506

Theory: 3 credits and Practicals: 1 credits
Theory: 3 hours/week and Practicals: 2 hours/week

Objective: Techniques of multiple integrals will be taught.

Outcome: Students will come to know about its applications in finding areas and volumes of some solids.

UNIT-I

Areas and Volumes: Double Integrals-Double Integrals over a Rectangle-Double Integrals over General Regions in the Plane.

UNIT-II

Double integrals, Changing the order of Integration, Triple Integrals: The Integrals over a Box.

UNIT-III

Elementary Regions in Space-Triple Integrals in General, Triple Integral.

UNIT-IV

Change of Variables: Coordinate Transformations-Change of Variables in Triple Integrals.

TEXT: Susan Jane Colley, *Vector Calculus*(4e)

References

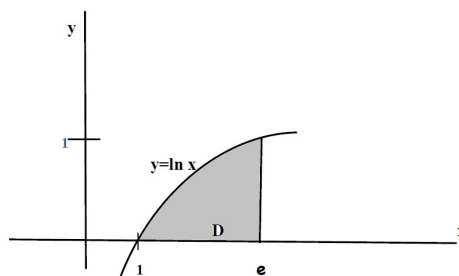
- Smith and Minton, *Calculus*
- Shanti Narayan and Mittal, *Integral calculus*
- Ulrich L. Rohde , G. C. Jain, Ajay K. Poddar and A. K. Ghosh; *Introduction to Integral Calculus*

2.14.1 Practicals Question Bank
UNIT-I

- (1) Let $R = [-3, 3] \times [-2, 2]$. Without explicitly evaluating any iterated integrals, determine the value of $\iint_R (x^5 + 2y) dA$.
- (2) Integrate the function $f(x, y) = 3xy$ over the region bounded by $y = 32x^3$ and $y = \sqrt{x}$.
- (3) Integrate the function $f(x, y) = x + y$ over the region bounded by $x + y = 2$ and $y^2 - 2y - x = 0$.
- (4) Evaluate $\iint_D xy dA$, where D is the region bounded by $x = y^3$ and $y = x^2$.
- (5) Evaluate $\iint_D e^{x^2} dA$, where D is the triangular region with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.
- (6) Evaluate $\iint_D 3y dA$, where D is the region bounded by $xy^2 = 1$, $y = x$, $x = 0$ and $y = 3$.
- (7) Evaluate $\iint_D (x - 2y) dA$, where D is the region bounded by $y = x^2 + 2$ and $y = 2x^2 - 2$.
- (8) Evaluate $\iint_D (x^2 + y^2) dA$, where D is the region in the first quadrant bounded by $y = x$, $y = 3x$ and $xy = 3$.
- (9) Let D be the region bounded by the parabolas $y = 3x^2$, $y = 4 - x^2$ and the y -axis (Note that parabolas intersect at the point $(1, 3)$). Since D is the type I elementary region, with $f(x, y) = x^2y$ then find $\iint_D x^2y dA = \int_0^1 \int_{3x^2}^{4-x^2} x^2y dy dx$
- (10) Find the volume of the region under the graph of $f(x, y) = 2 - |x| - |y|$ and above the xy -plane.

Unit-II

- (11) Calculate area of shaded region from the given figure. Consider D as type-I region



- (12) Use change of order of the integration find integral $\int_0^2 \int_{y^2}^4 y \cos(x^2) dx dy$.
- (13) consider the integral $\int_0^2 \int_{x^2}^{2x} (2x + 1) dy dx$ (a) Evaluate this integral. (b) Sketch the region of integration. (c) Write an equivalent iterated integral with the order of integration reversed. Evaluate this new integral and check that your answer agrees with part (a).
- (14) Evaluate $\iiint_{[-2,3] \times [0,1] \times [0,5]} (x^2e^y + xyz) dV$

- (15) Evaluate $\iiint_{[-1,1] \times [0,2] \times [1,3]} xyz dV$
- (16) Evaluate $\iiint_{[0,1] \times [0,2] \times [0,3]} (x^2 + y^2 + z^2) dV$
- (17) Evaluate $\iiint_{[1,e] \times [1,e] \times [1,e]} \frac{1}{xyz} dV$
- (18) Find the value of $\iiint_W z dV$, where $W = [-1, 2] \times [2, 5] \times [-3, 3]$, without resorting to explicit calculation.
- (19) Evaluate the iterative integral. $\int_{-1}^2 \int_1^{z^2} \int_0^{y+z} 3yz^2 dx dy dz$.
- (20) Evaluate the iterative integral. $\int_1^3 \int_0^z \int_1^{xz} (x + 2y + z) dy dx dz$.

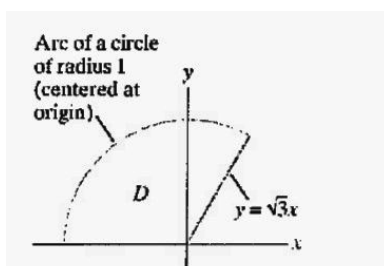
Unit-III

- (21) Let W be the solid region bounded by the hemisphere $x^2 + y^2 + z^2 = 4$ where $z \leq 0$ and the paraboloid $z = 4 - x^2 - y^2$ put solid bounded by them in type1,type2,type3,type4 forms and discuss the same geometrically.
- (22) Put the solid bounded by the ellipsoid $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, a, b, c are positive constants in the in type1,type2,type3,type4 forms and discuss the same geometrically.
Integrate the following over the indicated W .
- (23) $f(x, y, z) = 2x - y + z$; W is the region bounded by the cylinder $z = y^2$, the xy -plane, the planes $x = 0, x = 1, y = -2, y = 2$.
- (24) $f(x, y, z) = y$; W is the region bounded by the plane $x + y + z = 2$, the cylinder $x^2 + z^2 = 1$ and $y = 0$.
- (25) $f(x, y, z) = 8xyz$; W is the region bounded by the cylinder $y = x^2$, the plane $y + z = 9$ and the xy -plane.
- (26) $f(x, y, z) = z$; W is the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes $y = x, x = 0$ and $z = 0$.
- (27) $f(x, y, z) = 1 - z^2$; W is the tetrahedron with vertices $(0, 0, 0), (1, 0, 0), (0, 2, 0)$ and $(0, 0, 3)$.
- (28) $f(x, y, z) = 3x$; W is the region in the octant bounded by $z = x^2 + y^2, x = 0, y = 0$ and $z = 4$.
- (29) $f(x, y, z) = x + y$; W is the region bounded by the cylinder $x^2 + 3z^2 = 9$ and the plane $y = 0, x + y = 3$.
- (30) $f(x, y, z) = z$; W is the region bounded by $z = 0, x^2 + 4y^2 = 4$ and $z = x + 2$.

Unit-IV

- (31) Let $T : R^3 \rightarrow R^3$ be given by $T(u, v, w) = (2u, 2u + 3v + w, 3w)$ write T by matrix multiplication. Integrate the following over the indicated region W .
- (32) $f(x, y, z) = 4x + y$; W is the region bounded by $x = y^2, y = z, x = y$ and $z = 0$.
- (33) $f(x, y, z) = x$; W is the region in the first octant bounded by $z = x^2 + 2y^2, z = 6 - x^2 - y^2, x = 0$ and $y = 0$.

- (34) Let $T(u, v) = (3u, -v)$. Write $T(u, v)$ as $A[y]$ for a suitable matrix A .
- (35) Describe the image $D = T(D^*)$, where D^* is the unit square $[0, 1] \times [0, 1]$.
- (36) Determine the value of $\iint_D \sqrt{\frac{x+y}{x-2y}} dA$, where D is the region in R^2 enclosed by the lines $y = x^2$, $y = 0$ and $x + y = 1$.
- (37) Evaluate $\iint_D \sqrt{\frac{(2x+y-3)^2}{(2y-x+6)^2}} dx dy$, where D is the square with vertices $(0, 0)$, $(2, 1)$, $(3, -1)$ and $(1, -2)$. (Hint: First sketch D and find the equations of its sides).
- (38) Evaluate $\iint_D \cos(x^2 + y^2) dA$ where D is the shaded region in the following figure.



- (39) Evaluate $\iint_D \frac{1}{\sqrt{4-x^2-y^2}} dA$. where D is the disk of radius 1 with center at $(0, 1)$. (Be careful when you describe D .)
- (40) Determine the value of $\iiint_W \frac{z}{\sqrt{x^2+y^2}} dV$. where W is the solid region bounded by the plane $z = 12$ and the paraboloid $z = 2x^2 + 2y^2 - 6$.