2.2 Differential Equations

DSC-1B **BS:204**

Theory: 4 credits and Practicals: 1 credits
Theory: 4 hours /week and Practicals: 2 hours /week

Objective: The main aim of this course is to introduce the students to the techniques of solving differential equations and to train to apply their skills in solving some of the problems of engineering and science.
Outcome: After learning the course the students will be equipped with the various tools to solve few types differential equations that arise in several branches of science.

Unit- I

Differential Equations of first order and first degree: Exact differential equations - Integrating Factors - Change in variables - Total Differential Equations - Simultaneous Total Differential Equations - Equations of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Differential Equations first order but not of first degree: Equations Solvable for y - Equations Solvable for x - Equations that do not contain x (or$y$)- Clairaut's equation.

Unit- II

Higher order linear differential equations: Solution of homogeneous linear differential equations with constant coefficients - Solution of non-homogeneous differential equations $P(D)y = Q(x)$ with constant coefficients by means of polynomial operators when $Q(x) = b\mathrm{e}^{ax}, b\sin ax/b\cos ax, bx^k, V\mathrm{e}^{ax}$.

Unit- III

Method of undetermined coefficients - Method of variation of parameters - Linear differential equations with non constant coefficients - The Cauchy - Euler Equation.

Unit- IV

Partial Differential equations- Formation and solution- Equations easily integrable - Linear equations of first order - Non linear equations of first order - Charpit's method - Homogeneous linear partial differential equations with constant coefficient - Non homogeneous linear partial differential equations - Separation of variables.

Text:

- Zafar Ahsan,*Differential Equations and Their Applications*

References:

- Frank Ayres Jr, *Theory and Problems of Differential Equations.*
- Ford, L.R ; *Differential Equations.*
- Daniel Murray, *Differential Equations.*
- S. Balachandra Rao, *Differential Equations with Applications and Programs.*
- Stuart P Hastings, J Bryce McLead; *Classical Methods in Ordinary Differential Equations.*

2.2.1 Practicals Question Bank

Differential Equations

Unit-I

Solve the following differential equations:

1. $y\prime = \sin(x+y) + \cos(x+y)$
2. $xdy - ydx = a(x^2+y^2)dy$
3. $x^2ydx - (x^3+y^3)dy = 0$
4. $(y+z)dx + (x+z)dy + (x+y)dz = 0$
5. $y \sin 2xdx - (1+y^2+\cos^2 x)dy = 0$
6. $y + px = p^2x^4$
7. $yp^2 + (x-y)p - x = 0$
8. $\frac{dx}{y-zx} = \frac{dy}{yz+x} = \frac{dz}{(x^2+y^2)}$
9. $\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$
10. Use the transformation $x^2 = u$ and $y^2 = v$ to solve the equation $axyp^2 + (x^2 - ay^2 - b)p - xy = 0$

Unit-II

Solve the following differential equations:

11. $D^2y + (a+b)Dy + aby = 0$
12. $D^3y - D^2y - Dy - 2y = 0$
13. $D^3y + Dy = x^2 + 2x$
14. $y\prime\prime + 3y\prime + 2y = 2(e^{-2x} + x^2)$
15. $y^{(5)} + 2y\prime\prime\prime + y\prime = 2x + \sin x + \cos x$
16. $(D^2+1)(D^2+4)y = \cos\frac{x}{2}\cos\frac{3x}{2}$
17. $(D^2+1)y = \cos x + xe^{2x} + e^x \sin x$
18. $y\prime\prime + 3y\prime + 2y = 12e^x$
19. $y\prime\prime - y = \cos x$
20. $4y\prime\prime - 5y\prime = x^2e^x$

Unit-III

Solve the following differential equations:

21. $y'' + 3y' + 2y = xe^x$
22. $y'' + 3y' + 2y = \sin x$
23. $y'' + y' + y = x^2$
24. $y'' + 2y' + y = x^2 e^{-x}$
25. $x^2 y'' - xy' + y = 2\log x$
26. $x^4 y''' + 2x^3 y'' - x^2 y' + xy = 1$
27. $x^2 y'' - xy' + 2y = x\log x$
28. $x^2 y'' - xy' + 2y = x$

Use the reduction of order method to solve the following homogeneous equation whose one of the solution is given:

29. $y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 0, y_1 = x$
30. $(2x^2 + 1)y'' - 4xy' + 4y = 0, y_1 = x$

Unit-IV

31. Form the partial differential equation, by eliminating the arbitrary constants from $z = (x^2 + a)(y^2 + b)$.
32. Find the differential equation of the family of all planes whose members are all at a constant distance r from the origin.
33. Form the differential equation by eliminating arbitrary function F from $F(x^2 + y^2, z - xy) = 0$.

Solve the following differential equations:

34. $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$
35. $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$
36. $(p^2 - q^2)z = x - y$
37. $z = px + qy + p^2 q^2$
38. $z^2 = pqxy$
39. $z^2(p^2 + q^2) = x^2 + y^2$
40. $r + s - 6t = \cos(2x + y)$
