D.O.No. F. 1-1/2014(Sency)  

12th November, 2014

Dear Sir/Madam,

The UGC has embarked on numerous measures to enhance efficiency and excellence in the higher education system in the country. The reforms undertaken in this regard have led to noticeable improvement in the standards of education. However, because of the diversity in the evaluation system followed by different universities in India, students have suffered acceptance of their credentials, at times, across the university system, as well as the employment agencies.

In order to mitigate this procedure, it has been thought that the Choice-Based Credit System (CBCS) proposed by the UGC should be adopted by all the Universities. This would ensure seamless mobility of students across the higher education institutions in the country as well as abroad. The credits earned by the student can be transferred and would be of great value to the students in the event of their seeking migration from one institution to the other.

Even in the universities which have already adopted the CBCS it has come to our notice that there is tremendous diversity in the adoption of the system that inter-university migration of students amongst such universities has also posed problems. Under the situation mentioned, the UGC has formulated Guidelines for adoption of uniform Choice-Based Credit System across all the universities. The Guidelines have been uploaded on the website of the UGC (www.ugc.ac.in).

You are requested that the Guidelines may kindly be accessed from the UGC website and the system introduced in your esteemed university from the academic year 2015-16. All the actions taken in this regard may kindly be communicated to the Secretary, UGC (email: ugc.action@gmail.com).

With kind regards,

Yours sincerely,

(Jaspal S. Sandhu)

The Vice-Chancellors of all Universities.
Contents

1. Syllabus: Theory and Practicals

2. MOOCs (Massive Online Open Courses)

Resources for ICT based Learning and Teaching

3. Appendix 1

4. Appendix 2
Objective: The course is aimed at exposing the students to some basic notions in differential calculus.

Outcome: By the time students completes the course they realize wide ranging applications of the subject.

Unit I
Successive differentiation - Expansions of Functions - Mean value theorems

Unit II
Indeterminate forms - Curvature and Evolutes

Unit III
Partial differentiation - Homogeneous functions - Total derivative

Unit IV
Maxima and Minima of functions of two variables - Lagrange’s Method of multipliers - Asymptotes - Envelopes

Text: Shanti Narayan and Mittal, Differential Calculus

References: William Anthony Granville, Percey F Smith and William Raymond Longley, Elements of the differential and integral calculus

Joseph Edwards, Differential calculus for beginners

Smith and Minton, Calculus

Elis Pine, How to Enjoy Calculus

Hari Kishan, Differential Calculus
UNIT-I

1. If \( u = \tan^{-1} x \), prove that
\[
(1 + x^2) \frac{d^2u}{dx^2} + 2x \frac{du}{dx} = 0
\]
and hence determine the values of the derivatives of \( u \) when \( x = 0 \).

2. If
\[
y = \sin (m \sin^{-1} x),
\]
show that
\[
(1 - x^2)y_n + x = (2m + 1)xy_n + (n^2 - m^2)y_n
\]
and find \( y_n(0) \).

3. If \( U_n \) denotes the \( n \)-th derivative of \((Lx + M)/(x^4 - 2Bx + C)_r\), prove
\[
x^3 - 2Bx + C, \quad \frac{U_n}{(n+1)(n+2)} - 2x - B \quad \frac{U_{n+1}}{n+1} - U_n = 0.
\]

4. If \( y = x^m e^x \), then
\[
\frac{d^n y}{dx^n} = \frac{1}{2} n (n-1) \cdots (n-m) \frac{dy}{dx} + \frac{1}{2} (n-2)(n-2)y.
\]

5. Determine the intervals in which the function
\[
(x^4 + 6x^3 + 17x^2 + 32x + 32)e^{-x}
\]
is increasing or decreasing.

6. Separate the intervals in which the function
\[
(x^4 + x + 1)/(x^4 - x + 1)
\]
is increasing or decreasing.

7. Show that if \( x > 0 \),
\[
(i) \quad x - \frac{x^3}{2} < \log (1 + x) < x - \frac{x^3}{2(1 + x)}.
(ii) \quad x - \frac{x^3}{2} + \frac{x^5}{3(1 + x)} < \log (1 + x) < x - \frac{x^3}{2} + \frac{x^5}{3}.
\]

8. Prove that
\[
e^{ax} \sin bx = b \sin x + absx^2 + \frac{3a^2 b^2 - b^3}{3!} x^3 + \ldots
\]
\[
+ \frac{(a^2 + b^2)^{\frac{1}{2}}}{n!} x^n \sin \left( n \tan^{-1} \frac{b}{a} \right) + \ldots
\]

9. Show that \( \cos^2 x = 1 - x^2 + \frac{1}{3} x^4 - \ldots \)

10. Show that
\[
e^{m \tan^{-1} x} = 1 + mx + \frac{m^2}{2} x^2 + \frac{m(m^2 - 2)}{3!} x^3 + \frac{m^4(m^2 - 8)}{4!} x^4 + \ldots
\]

UNIT-II

1. Find the radius of curvature at any point on the curve
\((i) \ y = e \cosh (x/c) \) (Catenary).
\((ii) \ x = a \cos t - t \cos a \), \( y = a \sin t - t \sin a \).
\((iii) \ x = \frac{a}{2} + \frac{a^2}{4} \), \( y = \frac{a^2}{2} \) \) (Astroid).
\((iv) \ x = a \cos t, y = a \sin t \).

4. Show that for the curve
\( x = a \cos \theta \), \( y = a \sin \theta \),
the radius of curvature is, \( a \), at the point for which the value of the parameter
is \(-\pi/4\).

7. Prove that the radius of curvature at the point
\((-2a, 2a)\) on the curve \( x^2y - a(x^2 + y^2) \) is, \(-2a\).
3. Show that the radii of curvature of the curve
\[ x = a \theta (\sin \theta - \cos \theta), \quad y = a \theta (\sin \theta + \cos \theta), \]
and its evolute at corresponding points are equal.

4. Show that the whole length of the evolute of the ellipse
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]
is \(4(a^2/b - b^2/a)\).

5. Show that the whole length of the evolute of the astroid
\[ x = a \cos^2 \theta, \quad y = a \sin^2 \theta \]
is \(12a\).

Evaluate the following:

6. \(i\) \(\lim_{x \to 0} \frac{\tan^2 x - \log (1 + x)}{x^3} \), \(\lim_{x \to 0} \frac{x \cos x - \log (1 + x)}{x^4} \).

7. \(iii\) \(\lim_{x \to 0} \frac{e^x \sin x - x - x^2}{x^3 + x \log (1 - x)} \), \(\lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{x^2} \log (1 + x) \right) \).

8. If the limit of
\[ \frac{\sin 2x + a \sin x}{x^2} \]
as \(x\) tends to zero, be finite, find the value of \(a\) and the limit.

9. Determine the limits of the following functions:
\(i\) \(x \log \tan x, \quad (x \to 0)\), \(x \tan (\pi/2 - x), \quad (x \to 0)\).
\(ii\) \((a-x) \tan (nx/2a), \quad (x \to 0)\).

10. Determine the limits of the following functions:
\(\frac{e^x - e^{-x} - x}{x^3 \sin x}, \quad (x \to 0)\), \(\frac{\log x}{x^3}, \quad (x \to \infty)\),
\(\frac{1 + x \cos x - \cosh x - \log (1 + x)}{\tan x - x}, \quad (x \to 0)\),
\(\frac{\log (1 + x) \log (1 - x) - \log (1 - x^2)}{x^2}, \quad (x \to 0)\).

UNIT III

1. If \(z = xy f(x/y)\), show that
\[ x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 2z. \]

2. If \(z(x+y) = x^2 + y^2\), show that
\[ \left( \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2 = 4 \left( 1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right). \]

3. If \(z = 3xy - y^3 + (y^3 - 2x)^2\), verify that
\[ \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} \quad \text{and} \quad \frac{\partial^2 z}{\partial x^2} \quad \text{and} \quad \frac{\partial^2 z}{\partial y^3} = \left( \frac{\partial^2 z}{\partial x \partial y} \right)^2. \]

4. If \(z = f(x \cdot ay) + \varphi(x - ay)\), prove that
\[ \frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}. \]

5. If \(u = \tan^{-1} \frac{y^2 + x^2}{x - y}\), find
\[ \frac{x \partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}. \]
If \(f(x, y) = 0, \quad \varphi(y, z) = 0\), show that
\[ \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \quad \frac{\partial}{\partial \varphi} \quad \frac{\partial}{\partial \varphi} \quad \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \quad \frac{\partial}{\partial \varphi}. \]
7. If \[ x^4(1 - y^4) + y^4(1 - x^4) = a, \] show that
\[
\frac{dy}{dx} = \frac{a}{(1 - x^4)^{\frac{5}{2}}}.
\]

8. Given that
\[ f(x, y) = x^4 + y^3 - 3axy = 0, \]
show that
\[
\frac{d^3y}{dx^3} - \frac{4a^2}{x^2} \frac{dy}{dx} = xy(x^2 - 2a^4)^2.
\]

9. If \( u \) and \( v \) are functions of \( x \) and \( y \) defined by
\[ x = u + e^{-v} \sin u, \quad y = v + e^{-v} \cos u, \]
prove that
\[
\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}.
\]

10. If \( H = f(y - z, z - x, x - y) \); prove that,
\[
\frac{\partial^2 H}{\partial x \partial y} - \frac{\partial^2 H}{\partial y \partial z} = 0.
\]

UNIT-IV
1. Find the minimum value of \( x^2 + y^2 + z^2 \) when
   (i) \( x + y + z = 3a \).
   (ii) \( xy + yz + zx = 3a^2 \).
   (iii) \( xyz = a^3 \).

2. Find the extreme value of \( xy \) when
   \[ x^4 + xy + y^3 = a^3. \]

3. In a plane triangle, find the maximum value of
   \[ \cos A \cos B \cos C. \]

4. Find the envelope of the family of semi-cubical parabolae
   \[ y^3 = (x + a)^3 = 0. \]

5. Find the envelope of the family of ellipses
   \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \]
   where the two parameter \( a, b \), are connected by the relation
   \[ a + b = c; \]
   \( c \), being a constant.

6. Show that the envelope of a circle whose centre lies on the
   parabola \( y^2 = 4ax \) and which passes through its vertex is the cissoid
   \[ y^3(2a + x) + x^3 = 0. \]

7. Find the envelope of the family of straight lines \( x/a + y/b = 1 \) where
   \( a, b \) are connected by the relation
   (i) \( a + b = c \).
   (ii) \( a^2 + b^2 = c^2 \).
   (iii) \( ab = c^3 \),
   \( c \) is a constant.

8. Find the asymptotes of
   \[ x^2 + 4x^2y + 4xy^2 + 5x^2 + 15xy - 10y^2 - 2y + 1 = 0. \]

9. Find the asymptotes of
   \[ x^2 + 4x^2y + 4xy^2 + 5x^2 + 15xy - 10y^2 - 2y + 1 = 0. \]

10. Find the asymptotes of the following curves
    \[ xy(x + y) = a(x^2 - a^2), \]
    \[ (x - 1)(x - 2)(x + y) + x^2 + x + 1 = 0 \]
    \[ y^2 - x^2 + y^2 + x^2 + y^2 + x + 1 = 0 \]

302
Objective: The main aim of this course is to introduce the students to the techniques of solving differential equations and to train to apply their skills in solving some of the problems of engineering and science.

Outcomes: After learning the course the students will be equipped with the various tools to solve few types differential equations that arise in several branches of science.

Unit – I

Differential Equations of first order and first degree:

Exact differential equations – Integrating Factors – Change in variables – Total Differential Equations – Simultaneous Total Differential Equations – Equations of the form \( \frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \)

Differential Equations first order but not of first degree: Equations Solvable for \( y \) – Equations Solvable for \( x \) – Equations that do not contain \( x \) (or \( y \)) Clairaut’s equation

Unit – II

Higher order linear differential equations: Solution of homogeneous linear differential equations with constant coefficients – Solution of non-homogeneous differential equations \( P(D)y = Q(x) \) with constant coefficients by means of polynomial operators when \( Q(x) = bx^k, be^{ax}, e^{ax}, b \cos(ax), b \sin(ax) \)

Unit – III

Method of undetermined coefficients – Method of variation of parameters – Linear differential equations with non constant coefficients – The Cauchy Euler Equation
Unit – IV

Partial Differential equations- Formation and solution- Equations easily integrable -
Linear equations of first order – Non linear equations of first order – Charpit’s method
– Non homogeneous linear partial differential equations – Separation of variables

Text: Zafar Ahsan, Differential Equations and Their Applications

References: Frank Ayres Jr, Theory and Problems of Differential Equations

Ford, L.R, Differential Equations.

Daniel Murray, Differential Equations

S. Balachandra Rao, Differential Equations with Applications and Programs

Stuart P Hastings, J Bryce McLeod; Classical Methods in Ordinary Differential Equations
Unit-I

Solve the following differential equations:

1. \( y' = \sin(x + y) + \cos(x + y) \)

2. \( xdy - ydx = a(x^2 + y^2)dy \)

3. \( x^2ydx - (x^3 + y^3)dy = 0 \)

4. \( (y + z)dx + (x + z)dy + (x + y)dz = 0 \)

5. \( y\sin 2xdx - (1 + y^2 + \cos^2 x)dy = 0 \)

6. \( y + px = p^2x^4 \)

7. \( yp^2 + (x - y)p - x = 0 \)

8. \( \frac{dx}{y^2 - z^2} = \frac{dy}{yz + x} = \frac{dz}{x^2 + y^2} \)

9. \( \frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)} \)

10. Use the transformation \( x^2 = u \) and \( y^2 = v \) to solve the equation \( axyp^2 + (x^2 - ay^2 - b)p - xy = 0 \).

Unit-II

Solve the following differential equations:

1. \( D^2y + (a + b)Dy + aby = 0 \)

2. \( D^3y - D^2y - Dy - 2y = 0 \)

3. \( D^3y + Dy = x^2 + 2x \)

4. \( y'' + 3y' + 2y = 2(e^{-2x} + x^2) \)
5. \( y^{(5)} + 2y'' + y' = 2x + \sin x + \cos x \)

6. \((D^2 + 1)(D^2 + 4)y = \cos \frac{x}{2} \cos \frac{3x}{2}\)

7. \((D^2 + 1)y = \cos x + xe^{2x} + e^x \sin x \)

8. \( y'' + 3y' + 2y = 12e^x \)

9. \( y'' - y = \cos x \)

10. \( 4y'' - 5y' = x^2 e^x \)

Unit-III

Solve the following differential equations:

1. \( y'' + 3y' + 2y = xe^x \)

2. \( y'' + 3y' + 2y = \sin x \)

3. \( y'' + y' + y = x^2 \)

4. \( y'' + 2y' + y = x^2 e^{-x} \)

5. \( x^2 y'' - xy' + y = 2 \log x \)

6. \( x^4 y''' + 2x^3 y'' - x^2 y' + xy = 1 \)

7. \( x^2 y'' - xy' + 2y = x \log x \)

8. \( x^2 y'' - xy' + 2y = x \)

Use the reduction of order method to solve the following homogeneous equation whose one of the solutions is given:

9. \( y'' - \frac{2}{x} y' + \frac{2}{x^2} y = 0, \quad y_1 = x \)

10. \((2x^2 + 1)y'' - 4xy' + 4y = 0, \quad y_1 = x \)
Unit-IV

1. Form the partial differential equation, by eliminating the arbitrary constants from
   \[ z = (x^2 + a)(y^2 + b). \]

2. Find the differential equation of the family of all planes whose members are all at
   a constant distance \( r \) from the origin.

3. Form the differential equation by eliminating arbitrary function \( F \) from
   \[ F(x^2 + y^2, z - xy) = 0. \]

Solve the following differential equations:

4. \[ x^2(y - z)p + y^2(z - x)q = z^2(x - y) \]

5. \[ x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2) \]

6. \[ (p^2 - q^2)z = x - y \]

7. \[ z = px + qy + p^2q^2 \]

8. \[ z^2 = pqxy \]

9. \[ z^2(p^2 + q^2) = x^2 + y^2 \]

10. \[ r + s - 6t = \cos(2x + y) \]
LOGIC AND SETS

Credits: 2
Theory: 2 hours/week

Objective: Students learn some concepts in set theory and logic.
Outcome: After the completion of the course students appreciate its importance in the development of computer science.

Unit - I

Basic Connectives and truth tables - Logical equivalence - Laws of Logic - Logical Implication - Rules Inference - The Use of Quantifiers - Quantifiers, Definitions, and proofs of Theorems

Unit - II


Text: Ralph P Grimaldi, *Discrete and Combinatorial Mathematics (5e)*

References: P R Halmos, *Naive Set Theory*

E Kamke, *Theory of Sets*
Objective: Students learn the relation between roots and coefficients of a polynomial equation, Descartes’s rule of signs in finding the number of positive and negative roots if any of a polynomial equation besides some other concepts.

Outcome: By using the concepts learnt the students are expected to solve some of the polynomial equations.

Unit I

Graphic representation of a polynomial-Maxima and minima values of polynomials-Theorems relating to the real roots of equations-Existence of a root in the general equation-Imaginary roots-Theorem determining the number of roots of an equation-Equal roots-Imaginary roots enter equations in pairs-Descartes’ rule of signs for positive roots-Descartes’ rule of signs for negative roots-

Unit II

Relations between the roots and coefficients-Theorem-Applications of the theorem-Depression of an equation when a relation exists between two of its roots-The cube roots of unity-Symmetric functions of the roots-examples.

Text: W.S. Burnside and A.W. Panton, The Theory of Equations

References: C. C. Mac Duffee, Theory of Equations

Hall and Knight, Higher Algebra
Objective: The course is aimed at exposing the students to the foundations of analysis which will be useful in understanding various physical phenomena.

Outcome: After the completion of the course students will be in a position to appreciate beauty and applicability of the course.

Unit- I

Sequences- Limits of sequences- A Discussion about Proofs- Limit Theorems for Sequences – Monotone Sequences and Cauchy Sequences

Unit- II

Subsequences- Lim sup’s and Lim inf’s Series- Alternating Series and Integrals Tests. Continuity: Continuous functions- Properties of Continuous functions.

Unit – III

Sequence and Series of Functions: Power Series- Uniform Convergence – More on Uniform Convergence- Differentiation and Integration of Power Series (Theorems in this section without Proofs)

Unit – IV

Integration: The Riemann Integral- Properties of Riemann Integral- Fundamental Theorem of Calculus.

Text: Kenneth A Ross, Elementary Analysis- The Theory of Calculus

References:
William F. Trench: Introduction to Real Analysis
Lee Larson: Introduction to Real Analysis

Shanti Narayan and Mittal: Mathematical Analysis

Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner: Elementary Real Analysis

Sudhir R. Ghorpade Balmohan V. Limaye: A Course in Calculus and Real Analysis
Real Analysis
Practicals Question Bank

UNIT-I

1
For each sequence below, determine whether it converges and, if it converges, give its limit. No proofs are required.

(a) \( a_n = \frac{n}{n+1} \)
(b) \( b_n = \frac{n^2 + 1}{n^2 - 3} \)
(c) \( c_n = 2^{-n} \)
(d) \( t_n = 1 + \frac{2}{n} \)
(e) \( x_n = 73 + (-1)^n \)
(f) \( s_n = (2)^{1/n} \)

2
Determine the limits of the following sequences, and then prove your claims.

(a) \( a_n = \frac{n}{n+1} \)
(b) \( b_n = \frac{2n - 13}{3n + 7} \)
(c) \( c_n = \frac{4n + 3}{3n - 5} \)
(d) \( d_n = \frac{2n + 1}{5n + 2} \)
(e) \( s_n = \frac{1}{n} \sin n \)

3
Suppose \( \lim a_n = a, \lim b_n = b, \) and \( s_n = \frac{a_n + 4b_n}{b_n + 1} \). Prove \( \lim s_n = a^2 + 4b \) carefully, using the limit theorems.

4
Let \( x_1 = 1 \) and \( x_{n+1} = 3x_n^2 \) for \( n \geq 1 \).

(a) Show if \( a = \lim x_n \), then \( a = \frac{1}{3} \) or \( a = 0 \).

(b) Does \( \lim x_n \) exist? Explain.

(c) Discuss the apparent contradiction between parts (a) and (b)

5
Which of the following sequences are increasing, decreasing, bounded?

(a) \( \frac{1}{n} \)
(b) \( \frac{(1-n)^n}{n^3} \)
(c) \( n^n \)
(d) \( \sin(\frac{n\pi}{7}) \)
(e) \( (-2)^n \)
(f) \( \frac{n}{2^n} \)

6
Let \( (s_n) \) be a sequence such that

\[ |s_{n+1} - s_n| < 2^{-n} \]

for all \( n \in \mathbb{N} \).

Prove \( (s_n) \) is a Cauchy sequence and hence a convergent sequence.

7
Let \( (a_n) \) be an increasing sequence of positive numbers and define \( \sigma_n = \frac{1}{n}(a_1 + a_2 + \ldots + a_n) \). Prove \( (\sigma_n) \) is an increasing sequence.

8
Let \( t_1 = 1 \) and \( t_{n+1} = [1 - \frac{1}{2n^2}] \cdot t_n \) for \( n \geq 1 \).

(a) Show \( \lim t_n \) exists.

(b) What do you think \( \lim t_n \) is?
9
Let $t_1 = 1$ and $t_{n+1} = \left[1 - \frac{1}{(n+1)^2}\right] t_n$ for $n \geq 1$.

(a) Show $\lim t_n$ exists.
(b) What do you think $\lim t_n$ is?
(c) Use induction to show $t_n = \frac{n+1}{2n}$.
(d) Repeat part (b).

10
Let $s_1 = 1$ and $s_{n+1} = \frac{1}{2}(s_n + 1)$ for $n \geq 1$.

(a) Find $s_2$, $s_3$, and $s_4$.
(b) Use induction to show $s_n > \frac{1}{2}$ for all $n$.
(c) Show $(s_n)$ is a decreasing sequence.
(d) Show $\lim s_n$ exists and find $\lim s_n$.

UNIT II
11
Let $a_n = 3 + 2(-1)^n$ for $n \in \mathbb{N}$.

(a) List the first eight terms of the sequence $(a_n)$.
(b) Give a subsequence that is constant [takes a single value]. Specify the selection function $\sigma$.

12
Consider the sequences defined as follows:

\[ a_n = (-1)^n, \quad b_n = \frac{1}{n}, \quad c_n = n^2, \quad d_n = \frac{6n + 4}{7n - 3} \]

(a) For each sequence, give an example of a monotone subsequence.
(b) For each sequence, give its set of subsequential limits.
(c) For each sequence, give its $\limsup$ and $\liminf$.
(d) Which of the sequences converges? diverges to $+\infty$? diverges to $-\infty$?
(e) Which of the sequences is bounded?

13
Prove $\limsup |s_n| = 0$ if and only if $\lim s_n = 0$.

14
Let $(s_n)$ and $(t_n)$ be the following sequences that repeat in cycles of four:

\[ (s_n) = (0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, \ldots) \]
\[ (t_n) = (2, 1, 1, 0, 2, 1, 1, 0, 2, 1, 1, 0, 2, 1, 1, 0, \ldots) \]

Find

(a) $\lim \inf s_n + \lim \inf t_n$
(b) $\lim \inf (s_n + t_n)$
(c) $\lim \inf s_n + \lim \sup t_n$
(d) $\lim \sup (s_n + t_n)$
(e) $\lim \sup s_n - \lim \inf t_n$
(f) $\lim \sup (s_n - t_n)$
(g) $\lim \inf (s_n - t_n)$
Determine which of the following series converge. Justify your answers.

(a) \( \sum_{n=1}^{\infty} \left( \frac{2}{3} \right)^n \)
(b) \( \sum_{n=1}^{\infty} \frac{2^n}{n!} \)
(c) \( \sum_{n=1}^{\infty} \frac{2^n}{n^2} \)
(d) \( \sum_{n=1}^{\infty} \frac{3^n}{n!} \)
(e) \( \sum_{n=1}^{\infty} \frac{n^2}{n^3} \)
(f) \( \sum_{n=1}^{\infty} \frac{1}{n \log n} \)

16

Prove that if \( \sum a_n \) is a convergent series of nonnegative numbers and \( p > 1 \), then \( \sum a_n^p \) converges.

17

Show that if \( \sum a_n \) and \( \sum b_n \) are convergent series of nonnegative numbers, then \( \sum \sqrt{a_n b_n} \) converges. Hint: Show \( \sqrt{a_n b_n} \leq a_n + b_n \) for all \( n \).

18

We have seen that it is often a lot harder to find the value of an infinite sum than to show it exists. Here are some sums that can be handled.

(a) Calculate \( \sum_{n=1}^{\infty} \left( \frac{2}{3} \right)^n \) and \( \sum_{n=1}^{\infty} \left( -\frac{2}{3} \right)^n \).
(b) Prove \( \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1 \). Hint: Note that \( \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+1} \right) \).
(c) Prove \( \sum_{n=1}^{\infty} \frac{n}{2^n} = \frac{3}{2} \). Hint: Note \( \frac{k-1}{2^k} = \frac{k}{2^k} - \frac{k+1}{2^k} \).
(d) Use (c) to calculate \( \sum_{n=1}^{\infty} \frac{n}{2^n} \).

19

Determine which of the following series converge. Justify your answers.

(a) \( \sum_{n=2}^{\infty} \frac{1}{\sqrt{n \log n}} \)
(b) \( \sum_{n=2}^{\infty} \frac{\log n}{n} \)
(c) \( \sum_{n=1}^{\infty} \frac{1}{n \log n \log \log n} \)
(d) \( \sum_{n=2}^{\infty} \frac{\log n}{n^2} \)

20

Show \( \sum_{n=2}^{\infty} \frac{\log n}{n^p} \) converges if and only if \( p > 1 \).

UNIT-III

21

For each of the following power series, find the radius of convergence and determine the exact interval of convergence.

(a) \( \sum_{n=0}^{\infty} \left( \frac{2}{3} \right)^n x^n \)
(b) \( \sum_{n=0}^{\infty} \left( \frac{2}{3} \right)^n x^n \)
(c) \( \sum_{n=0}^{\infty} \left( \frac{1}{n^2} \right) x^n \)
(d) \( \sum_{n=0}^{\infty} \left( \frac{1}{n^2} \right) x^n \)
(e) \( \sum_{n=0}^{\infty} \left( \frac{1}{n!} \right) x^n \)
(f) \( \sum_{n=0}^{\infty} \left( \frac{1}{n!} \right) x^n \)

22

For \( r = 0, 1, 2, 3, \ldots \) let \( a_r = \left\lfloor \frac{1 + 2^{1/2} + 3^{1/3}}{r^{1/r}} \right\rfloor \).

(a) Find \( \limsup_{r \to \infty} a_r \) and \( \liminf_{r \to \infty} a_r \).
(b) Do the series \( \sum a_r \) and \( \sum \left( \frac{a_r}{a} \right) \) converge? Explain briefly.

23

Let \( r = 1, 2, 3, \ldots \). Prove that \( r^r \) converges uniformly to 0 on \( \mathbb{R} \).
Prove that if \( f_n \to f \) uniformly on a set \( S \), and if \( g_n \to g \) uniformly on \( S \), then \( f_n + g_n \to f + g \) uniformly on \( S \).

25

Let \( f_n(x) = \frac{x^n}{n} \). Show \( (f_n) \) is uniformly convergent on \([-1, 1]\) and specify the limit function.

26

Let \( f_n(x) = \frac{\sin(x)}{n} \) for all real numbers \( x \).

(a) Show \( (f_n) \) converges uniformly on \( \mathbb{R} \). \textit{Hint:} First decide what the limit function is, then show \( (f_n) \) converges uniformly to it.

(b) Calculate \( \lim_{n \to \infty} \int_{-2}^{2} f_n(x) \, dx \). \textit{Hint:} Don't integrate \( f_n \).

27

Show \( \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx \) converges uniformly on \( \mathbb{R} \) to a continuous function.

28

Show \( \sum_{n=1}^{\infty} \frac{x^n}{n^3} \) has radius of convergence 2 and the series converges uniformly to a continuous function on \([-2, 2]\).

29

(a) Show \( \sum_{n=1}^{\infty} \frac{x^n}{n^2} \) converges for \( x \in [0, 1) \).

(b) Show that the series converges uniformly on \([0, a]\) for each \( a, 0 < a < 1 \).

30

Suppose \( \sum_{k=1}^{\infty} g_k \) and \( \sum_{k=1}^{\infty} h_k \) converge uniformly on a set \( S \). Show \( \sum_{k=1}^{\infty} (g_k + h_k) \) converges uniformly on \( S \).

UNIT-IV

31

Let \( f(x) = x \) for rational \( x \) and \( f(x) = 0 \) for irrational \( x \).

(a) Calculate the upper and lower Darboux integrals for \( f \) on the interval \([0, b] \).

(b) Is \( f \) integrable on \([0, b] \)?

32

Let \( f \) be a bounded function on \([a, b]\). Suppose there exist sequences \((U_n)\) and \((L_n)\) of upper and lower Darboux sums for \( f \) such that \( \lim(U_n - L_n) = 0 \). Show \( f \) is integrable and \( \int_a^b f = \lim U_n = \lim L_n \).

33

A function \( f \) on \([a, b]\) is called a \textit{step function} if there exists a partition \( P = \{a = a_0 < a_1 < \cdots < a_m = b\} \) of \([a, b]\) such that \( f \) is constant on each interval \((a_{i-1}, a_i)\). Say \( f(x) = c_i \) for \( x \) in \((a_{i-1}, a_i)\).

(a) Show that a step function \( f \) is integrable and evaluate \( \int_a^b f \).

(b) Evaluate the integral \( \int_a^b P(x) \, dx \) for the postage-stamp function

34

Show \( \int_a^b \phi(x) \, dx \) converges.
Let \( f \) be a bounded function on \([a, b]\), so that there exists \( B > 0 \) such that \(|f(x)| \leq B\) for all \( x \in [a, b]\).

(a) Show

\[
U(f^2, P) - L(f^2, P) \leq 2B[U(f, P) - L(f, P)]
\]

for all partitions \( P \) of \([a, b]\). \textit{Hint:} \( f(x)^2 - f(y)^2 = [f(x) + f(y)] \cdot [f(x) - f(y)] \).

(b) Show that if \( f \) is integrable on \([a, b]\), then \( f^2 \) also is integrable on \([a, b]\).

36

Calculate

(a) \( \lim_{x \to 0} \frac{1}{x} \int_0^x e^{t^2} \, dt \) \quad \text{(b) } \lim_{h \to 0} \frac{1}{h} \int_3^{3+h} e^t \, dt \).

37

Show that if \( f \) is a continuous real-valued function on \([a, b]\) satisfying

\[
\int_a^b f(x)g(x) \, dx = 0 \text{ for every continuous function } g \text{ on } [a, b],
\]

then \( f(x) = 0 \) for all \( x \) in \([a, b]\).
Objective: Students learn Transportation problem, assignment problem Games with mixed strategies.
Outcome: Students come to know about nice applications of Operations Research.

Unit I

The Transportation and Assignment Problems: The Transportation Problem - A Streamlined Simplex Method for the Transportation Problem - The Assignment Problem

Unit II

Game Theory: The Formulation of Two-Person, Zero-Sum Games - Solving Simple Games—A Prototype Example - Games with Mixed Strategies - Graphical Solution Procedure - Solving by Linear Programming - Extensions

Text: Frederick S Hillier and Gerald J Lieberman. *An Elementary Introduction to Operations Research* (9e)


Gupta and Kapur; *Operations Research*
Objective: Students will be exposed to some of the jewels like Fermat's theorem, Euler's theorem in the number theory.
Outcome: Student uses the knowledge acquired solving some divisor problems.

Unit I

The Goldbach conjecture – Basic properties of congruences- Binary and Decimal Representation of Integers – Number Theoretic Functions: The Sum and Number of divisors- The Mobius Inversion Formula- The Greatest integer function

Unit II

Euler's generalization of Fermat's Theorem: Euler's Phi function- Euler's theorem-
Some Properties of the Euler's Phi function

Text: David M Burton, Elementary Number Theory (7e)

References: Thomas Koshy, Elementary Number Theory and its Applications
Kenneth H Rosen, Elementary Number Theory
Objective: The course is aimed at exposing the students to learn some basic algebraic structures like groups, rings etc.

Outcome: On successful completion of the course students will be able to recognize algebraic structures that arise in matrix algebra, linear algebra and will be able to apply the skills learnt in understanding various such subjects.

Unit – I

Groups: Definition and Examples of Groups - Elementary Properties of Groups - Finite Groups; Subgroups - Terminology and Notation - Subgroup Tests - Examples of Subgroups Cyclic Groups: Properties of Cyclic Groups – Classification of Subgroups Cyclic Groups - Permutation Groups: Definition and Notation - Cycle Notation - Properties of Permutations - A Check Digit Scheme Based on $D_5$

Unit – II

Isomorphisms; Motivation- Definition and Examples - Cayley’s Theorem Properties of Isomorphisms - Automorphisms-Cosets and Lagrange’s Theorem Properties of Cosets 138 | Lagrange’s Theorem and Consequences- An Application of Cosets to Permutation Groups - The Rotation Group of a Cube and a Soccer Ball - Normal Subgroups and Factor Groups: Normal Subgroups-Factor Groups - Applications of Factor Groups - Group Homomorphisms - Definition and Examples - Properties of Homomorphisms - The First Isomorphism Theorem

Unit III

Introduction to Rings: Motivation and Definition - Examples of Rings - Properties of Rings - Subrings - Integral Domains: Definition and Examples - Characteristics of a
Ring Ideals and Factor Rings; Ideals -Factor Rings -Prime Ideals and Maximal Ideals

Unit - IV

Ring Homomorphisms: Definition and Examples-Properties of Ring-Homomorphisms -The Field of Quotients Polynomial Rings: Notation and Terminology


Fraleigh, J.B. *A First Course in Abstract Algebra*.

Herstein, I.N. *Topics in Algebra*

Robert B. Ash, *Basic Abstract Algebra*

I Martin Isaacs, *Finite Group Theory*

Joseph J Rotman, *Advanced Modern Algebra*
1. Show that \{1, 2, 3\} under multiplication modulo 4 is not a group but that \{1, 2, 3, 4\} under multiplication modulo 5 is a group.

2. Let G be a group with the property that for any x, y, z in the group, \(xy = zx\) implies \(y = z\). Prove that G is Abelian.

3. Prove that the set of all \(3 \times 3\) matrices with real entries of the form
\[
\begin{pmatrix}
1 & a & b \\
0 & 1 & c \\
0 & 0 & 1
\end{pmatrix}
\]
is a group under multiplication.

4. Let G be the group of polynomials under addition with coefficients from \(\mathbb{Z}_{10}\). Find the orders of \(f(x) = 7x^2 + 5x + 4, g(x) = 4x^2 + 8x + 6\), and \(f(x) + g(x)\).

5. If \(a\) is an element of a group G and \(|a| = 7\), show that \(a\) is the cube of some element of G.

6. Suppose that \((a)\), \((b)\) and \((c)\) are cyclic groups of orders 6, 8, and 20, respectively. Find all generators of \((a)\), \((b)\), and \((c)\).

7. How many subgroups does \(\mathbb{Z}_{20}\) have? List a generator for each of these subgroups.

8. Consider the set \{4, 8, 12, 16\}. Show that this set is a group under multiplication modulo 20 by constructing its Cayley table. What is the identity element? Is the group cyclic? If so, find all of its generators.

9. Prove that a group of order 4 cannot have a subgroup of order 3.

10. Determine whether the following permutations are even or odd.
   a. (135)
   b. (1356)
   c. (13567)
   d. (12)(134)(152)
   e. (1243)(3521).

Unit-II

1. Show that the mapping \(a \to \log_{10} a\) is an isomorphism from \(\mathbb{R}^+\) under multiplication to \(\mathbb{R}\) under addition.

2. Show that the mapping \(f(a + bi) = a + bi\) is an automorphism of the group of complex numbers under addition.

3. Find all of the left cosets of \(\{1, 11\}\) in \(\mathbb{Z}/(20)\).
4. Let $C^*$ be the group of nonzero complex numbers under multiplication and let $H = \{a + bi \in C^*/a^2 + b^2 = 1\}$. Give a geometric description of the coset $(3 + 4i)H$. Give a geometric description of the coset $(c + di)H$.

5. Let $H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R}, ad \neq 0 \right\}$. Is $H$ a normal subgroup of $GL(2, \mathbb{R})$?

6. What is the order of the factor group $\mathbb{Z}/5\mathbb{Z}$?

7. Let $G = U(16), H = \{1, 15\}$, and $K = \{1, 9\}$. Are $H$ and $K$ isomorphic? Are $G/H$ and $G/K$ isomorphic?

8. Prove that the mapping from $R$ under addition to $GL(2, R)$ that takes $x$ to

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

is a group homomorphism. What is the kernel of the homomorphism?

9. Suppose that $f$ is a homomorphism from $\mathbb{Z}/30$ to $\mathbb{Z}/30$ and $\text{Ker} f = \{0, 10, 20\}$. If $f(23) = 9$, determine all elements that map to 9.

10. How many Abelian groups (up to isomorphism) are there?
   a. of order 6?
   b. of order 15?
   c. of order 42?
   d. of order $pq$, where $p$ and $q$ are distinct primes?
   e. of order $pqr$, where $p$, $q$, and $r$ are distinct primes?

Unit-III

1. Let $M_2(\mathbb{Z})$ be the ring of all $2 \times 2$ matrices over the integers and let $R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z} \right\}$.
   Prove or disprove that $R$ is a subring of $M_2(\mathbb{Z})$.

2. Suppose that $a$ and $b$ belong to a commutative ring $R$ with unity. If $a$ is a unit of $R$ and $b^2 = 0$, show that $a + b$ is a unit of $R$.

3. Let $n$ be an integer greater than 1. In a ring in which $x^n = x$ for all $x$, show that $ab = 0$ implies $ba = 0$.

4. List all zero-divisors in $\mathbb{Z}_{20}$. Can you see a relationship between the zero-divisors of $\mathbb{Z}_{20}$ and the units of $\mathbb{Z}_{20}$?

5. Let $a$ belong to a ring $R$ with unity and suppose that $a^n = 0$ for some positive integer $n$. (Such an element is called nilpotent.) Prove that $1 - a$ has a multiplicative inverse in $R$.

6. Let $d$ be an integer. Prove that $\mathbb{Z}[\sqrt{d}] = \{a + b\sqrt{d} : a, b \in \mathbb{Z}\}$ is an integral domain.

7. Show that $\mathbb{Z}[\sqrt{2}]$ has a nonzero nilpotent element if and only if $\sqrt{2}$ is divisible by the square of some prime.

   For all nonzero nilpotent elements $a$ and $b$, prove that $ab$ and $ba$ are nilpotent elements of $\mathbb{Z}[\sqrt{2}]$. 

Page 2
9. Find all maximal ideals in
   a. $\mathbb{Z}_8$
   b. $\mathbb{Z}_{10}$
   c. $\mathbb{Z}_{12}$
   d. $\mathbb{Z}_n$

10. Show that $R[x]/(x^2 + 1)$ is a field.

Unit-IV

1. Prove that every ring homomorphism $f$ from $\mathbb{Z}_n$ to itself has the form $f(x) = ax$, where $a^2 = a$.

2. Prove that a ring homomorphism carries an idempotent to an idempotent.

3. In $\mathbb{Z}$, let $A = \langle 2 \rangle$ and $B = \langle 8 \rangle$. Show that the group $A/B$ is isomorphic to the group $\mathbb{Z}_4$ but that the ring $A/B$ is not ring-isomorphic to the ring $\mathbb{Z}_4$.

4. Show that the number $9,897,654,527,609,805$ is divisible by $99$.

5. Show that no integer of the form $111,111,111,...,111$ is prime.

6. Let $f(x) = 4x^3 + 2x^2 + x + 3$ and $g(x) = 3x^4 + 3x^3 + 3x^2 + x + 4$, where $f(x), g(x) \in \mathbb{Z}_5[x]$. Compute $f(x) + g(x)$ and $f(x)g(x)$.

7. Let $f(x) = 5x^4 + 3x^3 + 1$ and $g(x) = 3x^2 + 2x + 1$ in $\mathbb{Z}_7[x]$. Determine the quotient and remainder upon dividing $f(x)$ by $g(x)$.

8. Let $f(x)$ belong to $\mathbb{Z}_p[x]$. Prove that if $f(b) = 0$, then $f(b^p) = 0$.

9. Determine which of the polynomials below is (are) irreducible over $\mathbb{Q}$.
   a. $x^3 + 9x^4 + 12x^2 + 6$
   b. $x^3 + x + 1$
   c. $x^4 + 3x^2 + 3$
   d. $x^5 + 5x^2 + 1$
   e. $(5/2)x^5 + (9/2)x^4 + 15x^3 + (3/7)x^2 + 6x + 3/14$.

10. Show that $x^2 + x + 4$ is irreducible over $\mathbb{Z}_{11}$. 

323
Objective: Students are exposed some basic ideas like random variables and its related concepts.
Outcome: Students will be able to their knowledge to solve some real world problems.

Unit I

Random Variables; Continuous Random Variables - Expectation of a Random Variable - Jointly Distributed Random Variables - Moment Generating Functions

Unit II


Text: Sheldon M Ross, *Introduction to Probability Models (9e)*

References: Miller and Miller, *Mathematical Statistics with Applications*

Hogg, McKean and Craig, *Introduction to Mathematical Statistics*

Gupta and Kapur, *Mathematical Statistics*
Credits: 2
Theory: 2 hours/week

Objective: Some of the Physics problems will be solved using Differential Equations.

Outcome: Student realizes some beautiful problems can be modeled by using differential equations.

Unit I
Linear Models-Nonlinear Models-Modeling with Systems of First-Order DEs-

Unit II
Linear Models: Initial-Value Problems-Spring/Mass Systems: Free Undamped Motion-
Spring/Mass Systems: Free Damped Motion-Spring/Mass Systems: Driven Motion-Series
Circuit Analogue-Linear Models: Boundary-Value Problems

Text: Dennis G Zill, *A first course in differential equations with modeling applications*

References: Shepley L. Ross, *Differential Equations*

I. Sneddon, *Elements of Partial Differential Equations*
LATTICE THEORY

Credits: 2  
Theory: 2 hours/week

Objective: Students will be exposed to Elements of theory of lattices.  
Outcome: Students apply their Knowledge problems on switching circuits.

Unit I

Lattices: Properties and Examples of Lattices - Distributive Lattices - Boolean 
Algebras - Boolean Polynomials - Ideals, Filters, and Equations - Minimal Forms of 
Boolean Polynomials

Unit II

Applications of Lattices - Switching Circuits - Applications of Switching Circuits - 
More Applications of Boolean Algebras


References: Davey and Priestly, *Introduction to Lattices and Order*
Objective: The students are exposed to various concepts like vector spaces, bases, dimension, Eigen values etc.

Outcome: After completion this course students appreciate its interdisciplinary nature.

Unit I

Vector Spaces: Vector Spaces and Subspaces - Null Spaces, Column Spaces, and Linear Transformations - Linearly Independent Sets: Bases - Coordinate Systems - The Dimension of a Vector Space

Unit II

Rank - Change of Basis - Eigenvalues and Eigenvectors - The Characteristic Equation

Unit III

Diagonalization - Eigenvectors and Linear Transformations - Complex Eigenvalues - Applications to Differential Equations - Orthogonality and Least Squares: Inner Product, Length, and Orthogonality - Orthogonal Sets

Text: David C Lay, Linear Algebra and its Applications 4e

References: S Lang, Introduction to Linear Algebra

Gilbert Strang, Linear Algebra and its Applications

Stephen H Friedberg et al, Linear Algebra

Kuldeep Singh, Linear Algebra

Sheldon Axler, Linear Algebra Done Right
UNIT-1
1

Let \( H \) be the set of all vectors of the form \( \begin{pmatrix} -2t \\ 4t \\ 3t \end{pmatrix} \). Find a vector \( v \) in \( \mathbb{R}^3 \) such that \( H = \text{Span} \{ v \} \). Why does this show that \( H \) is a subspace of \( \mathbb{R}^3 \)?

2

Let \( V \) be the first quadrant in the \( xy \)-plane; that is, let
\[
V = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x \geq 0, y \geq 0 \right\}
\]

a. If \( u \) and \( v \) are in \( V \), is \( u + v \) in \( V \)? Why?

b. Find a specific vector \( u \) in \( V \) and a specific scalar \( c \) such

3

Let \( v_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \) and \( v_2 = \begin{pmatrix} -2 \\ 7 \\ -9 \end{pmatrix} \). Determine if \( \{ v_1, v_2 \} \) is a basis for \( \mathbb{R}^3 \). Is \( \{ v_1, v_2 \} \) a basis for \( \mathbb{R}^2 \)?

4

The set \( B = \{ 1 + t^2, t + t^2, 1 + 2t + t^2 \} \) is a basis for \( \mathbb{P}_2 \). Find the coordinate vector of \( p(t) = 1 + 4t + 7t^2 \) relative to \( B \).

5

The set \( B = \{ 1 - t^2, t - t^2, 2 - t + t^2 \} \) is a basis for \( \mathbb{P}_2 \). Find the coordinate vector of \( p(t) = 1 + 3t - 6t^2 \) relative to \( B \).

6

The vectors \( v_1 = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} \), \( v_2 = \begin{pmatrix} 2 \\ -8 \\ -3 \end{pmatrix} \), \( v_3 = \begin{pmatrix} 7 \\ -7 \\ -7 \end{pmatrix} \) span \( \mathbb{R}^2 \) but do not form a basis. Find two different ways to express \( \begin{pmatrix} 1 \\ 1 \end{pmatrix} \) as a linear combination of \( v_1, v_2, v_3 \).

7

Find the dimension of the subspace of all vectors in \( \mathbb{R}^4 \) whose first and third entries are equal.

8

Find the dimension of the subspace \( H \) of \( \mathbb{R}^4 \) spanned by
\[
\begin{pmatrix} 1 \\ 5 \\ 10 \\ 18 \end{pmatrix}
\]

9

Let \( H \) be an \( n \)-dimensional subspace of an \( m \)-dimensional vector space \( V \). Show that \( H \neq V \).

10

Let \( H \) be an \( n \)-dimensional subspace of an \( m \)-dimensional vector space \( V \). Show that \( H \neq V \).
UNIT-II

11
If a $4 \times 7$ matrix $A$ has rank 3, find dim Nul $A$, dim Row $A$, and rank $A^T$.

12
If a $7 \times 5$ matrix $A$ has rank 2, find dim Nul $A$, dim Row $A$, and rank $A^T$.

13
If the null space of an $8 \times 5$ matrix $A$ is 3-dimensional, what is the dimension of the row space of $A^T$?

14
If $A$ is a $3 \times 7$ matrix, what is the smallest possible dimension of Nul $A$?

15
Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find $\mathbf{v} \in \mathbb{R}^4$ such that $\begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \end{bmatrix} = \mathbf{u} \mathbf{v}^T$.

16
If $A$ is a $7 \times 5$ matrix, what is the largest possible rank of $A^T$?
If $A$ is a $5 \times 7$ matrix, what is the largest possible rank of $A$?
Explain your answers.

17
Without calculations, list rank $A$ and dim Nul $A$

$$A = \begin{bmatrix} 2 & 6 & -6 & 6 & 3 & 6 \\ -2 & -3 & 6 & -3 & 0 & -6 \\ 4 & 9 & -12 & 9 & 3 & 12 \\ -2 & 7 & 6 & 3 & 3 & -6 \end{bmatrix}$$

18
Use a property of determinants to show that $A$ and $A^T$ have the same characteristic polynomial.

19
Find the characteristic equation of

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

20
Find the characteristic polynomial and the real eigenvalues of

$$\begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$
21
Let $A = PDP^{-1}$ and compute $A^4$

\[
\begin{bmatrix}
5 & 7 \\
2 & 3
\end{bmatrix}
\begin{bmatrix}
1 & 2 \\
2 & 3
\end{bmatrix}
\]

22
Let $B = \{b_1, b_2, b_3\}$ and $D = \{d_1, d_2\}$ be bases for vector spaces $V$ and $W$, respectively. Let $T : V \rightarrow W$ be a linear transformation with the property that

$T(b_1) = 3d_1 - 5d_2$, \quad $T(b_2) = -d_1 + 6d_2$, \quad $T(b_3) = 4d_2$.

Find the matrix for $T$ relative to $B$ and $D$.

23
Let $D = \{d_1, d_2\}$ and $B = \{b_1, b_2\}$ be bases for vector spaces $V$ and $W$, respectively. Let $T : V \rightarrow W$ be a linear transformation with the property that

$T(d_1) = 3b_1 - 3b_2$, \quad $T(d_2) = -2b_1 + 5b_2$.

Find the matrix for $T$ relative to $D$ and $B$.

24
Let $B = \{b_1, b_2, b_3\}$ be a basis for a vector space $V$ and let $T : V \rightarrow \mathbb{R}^2$ be a linear transformation with the property that

$T(\lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 b_3) = \begin{bmatrix} 2x_1 - 3x_2 + 3x_3 \\ -2x_1 + 5x_2 \end{bmatrix}$

Find the matrix for $T$ relative to $B$ and the standard basis for $\mathbb{R}^2$.

25
Let $T : P_2 \rightarrow P_2$ be the transformation that maps a polynomial $p(t)$ into the polynomial $(t + 3)p(t)$.

a. Find the image of $p(t) = 3 - 2t + t^2$.

b. Show that $T$ is a linear transformation.

c. Find the matrix for $T$ relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3\}$.

26
Assume the mapping $T : V \rightarrow V$ defined by

$T(\alpha_0 + \alpha_1 t + \alpha_2 t^2) = 3\alpha_0 + (15\alpha_0 - 2\alpha_1) t + (4\alpha_1 + \alpha_2) t^2$

is linear. Find the matrix representation of $T$ relative to the basis $B = \{1, t, t^2\}$.

27
Define $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ by $T(p)$

\[
\begin{bmatrix}
p(-2) \\
p(3) \\
p(1) \\
p(0)
\end{bmatrix}
\]

a. Show that $T$ is a linear transformation.

b. Find the matrix for $T$ relative to the basis $\{1, t, t^2, t^3\}$ for the standard basis for $\mathbb{R}^4$. 

330
Let \( A \) be a \( 2 \times 2 \) matrix with eigenvalues \(-3\) and \(-1\) and corresponding eigenvectors \( v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \) and \( v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \). Let \( X(t) \) be the position of a particle at time \( t \). Solve the initial value problem \( \dot{x} = Ax, \ x(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \).

Construct the general solution of \( \dot{x} = Ax \).

\[
A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}
\]

\[
= \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}
\]

Compute the orthogonal projection of \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) onto the line through \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) and the origin.
Unit I Theory: 3 credits and Practicals: 1 credit
Theory: 3 hours /week and Practicals: 2 hours /week

Objective: Students learn to describe some of the surfaces by using analytical geometry.

Outcome: Students understand the beautiful interplay between algebra and geometry.

Unit I
Spheres: Definition-The Sphere Through Four Given Points-Equations of a Circle-
Intersection of a Sphere and a Line-Equation of a Tangent Plane-Angle of Intersection
of Two Spheres-Radical Plane

Unit II
Cones and Cylinders: Definition-Condition that the General Equation of second
degree Represents a Cone-Cone and a Plane through its Vertex -Intersection of a Line
with a Cone- The Right Circular Cone-The Cylinder- The Right Circular Cylinder

Unit III
The Conicoid: The General Equation of the Second Degree-Intersection of Line with
a Conicoid-Plane of contact-Enveloping Cone and Cylinder

Text: Shanti Narayan and P K Mittal. Analytical Solid Geometry (17e)

References: Khaleel Ahmed. Analytical Solid Geometry

S L Loney. Solid Geometry

Smith and Minton. Calculus
Analytical Solid Geometry
Practicals Question Bank

UNIT-I

1. Find the equation of the sphere through the four points
   \((4, -1, 2), (0, -2, 3), (1, -5, -1), (2, 0, 1)\).

2. Find the equation of the sphere through the four points
   \((0, 0, 0), (-a, b, c), (a, -b, c), (0, b, -c)\).

3. Find the centre and the radius of the circle
   \(x^2 + y^2 + 2x = 16, \ x^2 + y^2 + 2y - 12 = 11\).

4. Show that the following points are concyclic:
   \((4, 6, 0, 2), (2, -0, 0, 7, -3, 8, -4, 0, 9)\).
   \((-3, 8, 5, 2.1, -5, 2, 2), -7, 0, 6, 1, -3, 4, 0)\).

5. Find the centres of the two spheres which touch the plane
   \(4x + 3y = 47\)
at the point \((8, 5, 4)\) and which touch the sphere
   \(x^2 + y^2 + z^2 = 1\).

6. Show that the spheres
   \(x^2 + y^2 + z^2 = 25\)
\(x^2 + y^2 + z^2 - 24x - 40y - 18z + 226 = 0\)
touch externally and find the point of the contact.

7. Find the equation of the sphere that passes through the two points
   \((0, 0, 0), (-5, -1, -4)\)
   and cuts orthogonally the two spheres
   \(x^2 + y^2 + z^2 - 2 = 0, 2(x^2 + y^2 + z^2) + z + 3y + 4 = 0\).

8. Find the limiting points of the co-axial system of spheres
   \(x^2 + y^2 + z^2 - 20x + 30y - 40z + 28 + \lambda(2x - 3y + 4z) = 0\).

9. Find the equations to the two spheres of the co-axial system
   \(x^2 + y^2 + z^2 - 6 - 6\lambda(2x + y + 3z - 3) = 0\),
which touch the plane
   \(3x + 4y = 15\).

10. Show that the radical planes of the sphere of a co-axial system and of any given sphere pass through a line.

UNIT-II

11. Find the equation of the cone whose vertex is the point \((1, 1, 0)\) and whose guiding curve is
    \(y = 0, x^2 + z^2 = 4\).

12. The section of a cone whose vertex is \(P\) and guiding curve the ellipse
    \(x^2/4 + y^2/4 = 1, z = 0\) by the plane \(x = 0\) is a rectangular hyperbola. Show that
    the locus of \(P\) is
    \(x^2 - y^2 + z^2 = 0\).

13. Find the enveloping cone of the sphere
    \(x^2 + y^2 + z^2 = 2x + 4z - 1\)
with its vertex at \((1, 1, 1)\).
Find the equation of the quadric cone whose vertex is at the origin and which passes through the curve given by the equations
\[ ax^2 + by^2 + cz^2 = 1, \quad lx + my + nz = p. \]

15
Find the equation of the cone with vertex at the origin and direction cosines of its generators satisfying the relation
\[ 3l^2 - 4m^2 + 5n^2 = 0. \]

16
Find the equation of the cylinder whose generators are parallel to
\[ x = -\frac{1}{2}y = \frac{1}{3}z; \]
and whose guiding curve is the ellipse
\[ x^2 + 2y^2 = 1, \quad z = 3. \]

17
Find the equation of the right circular cylinder of radius 2 whose axis is the line
\[ (x-1)/2 = (y-2)/3 = (z-3)/2. \]

18
The axis of a right circular cylinder of radius 2 is
\[ x = \frac{1}{2}, \quad y = \frac{3}{2}, \quad z = 3; \]
show that its equation is
\[ 10x^2 + 5y^2 + 13z^2 - 12xy - 6yz - 4x - 8y + 30y - 74z + 69 = 0. \]

19
Find the equation of the circular cylinder whose guiding circle is
\[ x^2 + y^2 + z^2 = 9, \quad x - y + z = 3. \]

20
Obtain the equation of the right circular cylinder described on the circle through the three points \((1, 0, 0), (0, 1, 0), (0, 0, 1)\) as guiding circle.

UNIT-III
21
Find the points of intersection of the line
\[ -\frac{1}{2}(x+5) = (y-4) = \frac{1}{2}(z-11) \]
with the conoid
\[ 12x^2 + 17y^2 + 7z^2 = 7. \]

22
Find the equations to the tangent planes to
\[ 7x^2 - 3y^2 = z^2 + 21 = 0, \]
which pass through the line,
\[ 7x - (y + 1) = 3, \quad z = 3. \]

23
Obtain the tangent planes to the ellipsoid
\[ x^2/a^2 + y^2/b^2 + z^2/c^2 = 1, \]
which are parallel to the plane
\[ lx + my + nz = 0. \]

24
Show that the plane \(3x + 12y - 8z - 17 = 0\) touches the conoid
\[ 3x^2 - 8y^2 + 9z^2 + 17 = 0, \]
and find the point of contact.
Find the equations to the tangent planes to the surface
\[ 4x^2 - 5y^2 + 7z^2 + 13 = 0, \]
parallel to the plane
\[ 4x + 20y - 21z = 0. \]
Find their points of contact also.

Find the locus of the perpendiculars from the origin to the tangent planes to the surface
\[ 12/x^2 + y^2/6^2 + z^2/5^2 = 1 \]
which cut off from its axes intercepts the sum of whose reciprocals is equal to a constant \(1/k\).

If the section of the enveloping cone of the ellipsoid
\[ x^2/a^2 + y^2/b^2 + z^2/c^2 = 1, \]
whose vertex is \(P\) by the plane \(z = 0\) is a rectangular hyperbola, show that the locus of \(P\) is
\[ x^2 + y^2 + z^2 \]
\[ a^2 + b^2 + c^2 = 1. \]

Find the locus of points from which three mutually perpendicular tangent lines can be drawn to the conicoid \(ax^2 + by^2 + cz^2 = 1\).

\(P(1, 3, 2)\) is a point on the conicoid,
\[ x^2 - 2y^2 + 3z^2 + 5 = 0. \]
Find the locus of the mid-points of chords drawn parallel to \(OP\).
Objective: Techniques of multiple integrals will be taught.

Outcome: Students will come to know about its applications in finding areas and volumes of some solids.

Unit I

Areas and Volumes: Double Integrals: Double Integrals over a Rectangle- Double Integrals over General Regions in the Plane-Changing the order of Integration

Unit II

Triple Integrals: The Integrals over a Box- Elementary Regions in Space-Triple Integrals in General

Unit III

Change of Variables: Coordinate Transformations-Change of Variables in Triple Integrals

Text: Susan Jane Colley, *Vector Calculus* (4e)

References: Smith and Minton, *Calculus*

Shanti Narayan and Mittal, *Integral Calculus*

Unit 1

1. Let \( R = [-3, 3] \times [-2, 2] \). Without explicitly evaluating any iterated integrals, determine the value of \( \iiint_{R} (x^3 + 2y) \, dA \).

2. Integrate the function \( f(x, y) = 3xy \) over the region bounded by \( y = 32x^4 \) and \( y = \sqrt{x} \).

3. Integrate the function \( f(x, y) = x + y \) over the region bounded by \( x + y = 2 \) and \( y^2 - 2y - x = 0 \).

4. Evaluate \( \iiint_{D} xy \, dA \), where \( D \) is the region bounded by \( x = y^3 \) and \( y = x^2 \).

5. Evaluate \( \iiint_{D} e^{x^2} \, dA \), where \( D \) is the triangular region with vertices \((0, 0), (1, 0), \) and \((1, 1)\). 

6. Evaluate \( \iiint_{D} 3y \, dA \), where \( D \) is the region bounded by \( xy^3 = 1 \), \( y = x \), \( x = 0 \), and \( y = 3 \).

7. Evaluate \( \iiint_{D} (x - 2y) \, dA \), where \( D \) is the region bounded by \( y = x^2 + 2 \) and \( y = 2x^2 - 2 \).

8. Evaluate \( \iiint_{D} (x^2 + y^2) \, dA \), where \( D \) is the region in the first quadrant bounded by \( y = x \), \( y = 3x \), and \( xy = 3 \).

9. Consider the integral

\[
\int_{0}^{2} \int_{x}^{2} (2x + 1) \, dy \, dx.
\]

a) Evaluate this integral.

b) Sketch the region of integration.

c) Write an equivalent iterated integral with the order of integration reversed. Evaluate this new integral and check that your answer agrees with part (a).

10. Find the volume of the region under the graph of

\( f(x, y) = 2 - |x| - |y| \)

and above the \( xy \)-plane.
Unit II

Integrate the following over the indicated region W.

11. \( f(x, y, z) = 2x - v + z; \ W \) is the region bounded by the cylinder \( z = v^2 \), the \( xy \)-plane, and the planes \( x = 0, z = 1, v = -2, y = 2 \).

12. \( f(x, y, z) = v; \ W \) is the region bounded by the plane \( x + y + z = 2 \), the cylinder \( y^2 + z^2 = 1 \), and \( y = 0 \).

13. \( f(x, y, z) = 8xy; \ W \) is the region bounded by the cylinder \( y = x^2 \), the plane \( y + z = 9 \), and the \( xy \)-plane.

14. \( f(x, y, z) = z; \ W \) is the region in the first octant bounded by the cylinder \( y^2 + z^2 = 9 \) and the planes \( v = x, x = 0, \) and \( z = 0 \).

15. \( f(x, y, z) = 1 - z^2; \ W \) is the tetrahedron with vertices \((0, 0, 0), (1, 0, 0), (0, 2, 0),\) and \((0, 0, 3)\).

16. \( f(x, y, z) = 3x; \ W \) is the region in the first octant bounded by \( z = x^2 + y^2, x = 0, v = 0, \) and \( z = 4 \).

17. \( f(x, y, z) = x + v; \ W \) is the region bounded by the cylinder \( x^2 + 2z^2 = 9 \) and the planes \( z = 0, x + v = 3 \).

18. \( f(x, y, z) = z; \ W \) is the region bounded by \( z = 0, v^2 + 4y^2 = 4, \) and \( z = x + 2 \).

19. \( f(x, y, z) = 4x + y; \ W \) is the region bounded by \( v = y^2, y = z, x = y, \) and \( z = 0 \).

20. \( f(x, y, z) = x; \ W \) is the region in the first octant bounded by \( z = x^2 + 2y^2, z = 6 - x^2 - y^2, v = 0, \) and \( y = 0 \).

Let \( T(u, v) = (3u, -v) \).

21. (a) Write \( T(u, v) \) as \( A \begin{bmatrix} u \\ v \end{bmatrix} \) for a suitable matrix \( A \).

22. (b) Describe the image \( D = T(D') \), where \( D \) is the unit square \([0, 1] \times [0, 1] \).

23. Determine the value of

\[
\int_{D} \sqrt{\frac{x + y}{x - 2y}} \, dA.
\]

where \( D \) is the region in \( \mathbb{R}^2 \) enclosed by the lines.
24. Evaluate
\[ \iint_D \frac{(2x + y - 3)^2}{(2y - x + 6)^2} \, dx \, dy. \]

where \( D \) is the square with vertices \((0, 0), (2, 1), (3, -1), \) and \((1, -2)\). (Hint: First sketch \( D \) and find the equations of its sides.)

25. Evaluate
\[ \iint_D \cos(x^2 + y^2) \, dA. \]

where \( D \) is the shaded region in Figure 5.106.

Arc of a circle of radius 1 (centered at origin)

The region \( D \) of Exercise 25.

26. Evaluate
\[ \iint_D \frac{1}{\sqrt{4 - x^2 - y^2}} \, dA. \]

where \( D \) is the disk of radius 1 with center at \((0, 1)\). Be careful when you describe \( D \).

27. Determine the value of \( \iiint_W \frac{z}{\sqrt{x^2 + y^2}} \, dV \), where \( W \) is the solid region bounded by the plane \( z = 12 \) and the paraboloid \( z = 2x^2 + 2y^2 - 6 \).
Objective: Students will be exposed to Elements of theory of lattices.
Outcome: Students apply their Knowledge problems on switching circuits.

Unit I

Lattices: Properties and Examples of Lattices - Distributive Lattices - Boolean Algebras - Boolean Polynomials - Ideals, Filters, and Equations - Minimal Forms of Boolean Polynomials

Unit II:

Applications of Lattices - Switching Circuits - Applications of Switching Circuits - More Applications of Boolean Algebras


References: Davey and Priestly, *Introduction to Lattices and Order*
GRAPH THEORY

Credits: 2
Theory: 2 hours/week

Objective: The students will be exposed to some basic ideas of group theory.
Outcome: Students will be able to appreciate the subject learnt.

Unit I

Graphs: A Gentle Introduction - Definitions and Basic Properties - Isomorphism

Unit II

Paths and Circuits: Eulerian Circuits - Hamiltonian Cycles - The Adjacency Matrix
Shortest Path Algorithms

Text: Edgar Goodaire and Michael M. Parmenter. Discrete Mathematics with Graph Theory (2e)

References: Rudolf Lidl and Gunter Pilz. Applied Abstract Algebra
S Pirzada, Introduction to Graph Theory
Objective: Students will be exposed to some elements of number theory.
Outcome: Students apply their Knowledge problems on check digits, modular designs.

Unit I

The Division Algorithm - Number Patterns - Prime and Composite Numbers - Fibonacci and Lucas' numbers - Fermat Numbers - GCD - The Euclidean Algorithm - The Fundamental Theorem of Arithmetic - LCM - Linear Diophantine Equations - Congruences - Linear Congruences

Unit II

The Pollard Rho Factoring Method - Divisibility Tests - Modular Designs - Check Digits - The Chinese Remainder Theorem - General Linear Systems - 2X2 Systems - Wilson's Theorem - Fermat's Little Theorem - Pseudo primes - Euler's Theorem

Text: Thomas Koshy, *Elementary Number Theory with Applications*

References: David M Burton, *Elementary Number Theory*
Objective: Students will be made to understand some methods of numerical analysis.

Outcome: Students realize the importance of the subject in solving some problems of algebra and calculus.

Unit - I


Unit - II

Interpolation and Polynomial Approximation: Interpolation and the Lagrange Polynomial - Data Approximation and Neville’s Method - Divided Differences - Hermite Interpolation - Cubic Spline Interpolation

Unit - III


References: M K Jain, S R K Iyengar and R k Jain, *Numerical Methods for Scientific and Engineering computation*

B. Bradie *A friendly introduction to Numerical Analysis*
UNIT-1

1 Use the Bisection method to find $p_1$ for $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$.

2 Let $f(x) = 3(x + 1)(x - \frac{1}{2})(x - 1)$. Use the Bisection method on the following intervals to find $p_1$.
   a. $[-2, 1.5]$   b. $[-1.25, 2.5]$  

3 Use the Bisection method to find solutions accurate to within $10^{-5}$ for the following problems.
   a. $x - 2^{-x} = 0$ for $0 \leq x \leq 1$
   b. $e^x - x^2 + 3x - 2 = 0$ for $0 \leq x \leq 1$
   c. $2x \cos(2x) - (x + 1)^2 = 0$ for $-3 \leq x \leq -2$ and $-1 \leq x \leq 0$

4 1. Use algebraic manipulation to show that each of the following functions has a fixed point at $p$ precisely when $f(p) = 0$, where $f(x) = x^4 + 2x^3 - x - 3$.
   a. $g_1(x) = (3 + x - 2x^2)^{1/4}$
   b. $g_2(x) = \left( \frac{x + 3 - 2x^4}{2} \right)^{1/2}$

5 Use a fixed-point iteration method to determine a solution accurate to within $10^{-5}$ for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

6 Use a fixed-point iteration method to determine a solution accurate to within $10^{-5}$ for $x^3 - x - 1 = 0$ on $[1, 2]$. Use $p_0 = 1$.

7 Use a fixed-point iteration method to find an approximation to $\sqrt{3}$ that is accurate to within $10^{-4}$.

8 The equation $x^3 - 10 \cos{x} = 0$ has two solutions, $\pm 1.3793646$. Use Newton's method to approximate the solutions to within $10^{-5}$ with the following values of $p_0$.
   a. $p_0 = -100$  b. $p_0 = -50$  c. $p_0 = -25$
   d. $p_0 = 25$  e. $p_0 = 50$  f. $p_0 = 100$

9 The equation $4x^4 - e^x - e^{-x} = 0$ has two positive solutions $x_1$ and $x_2$. Use Newton's method to approximate the solution to within $10^{-5}$ with the following values of $p_0$.

10 Use each of the following methods to find a solution in $[0.1, 1]$ accurate to within $10^{-4}$ for $600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0$.
   a. Bisection method  c. Secant method  e. Muller's method
   b. Newton's method  d. method of False Position
UNIT-II

11
For the given functions \( f(x) \), let \( x_0 = 0, x_1 = 0.6 \), and \( x_2 = 0.9 \). Construct interpolation polynomials of
degree at most one and at most two to approximate \( f(0.45) \), and find the absolute error.

a. \( f(x) = \cos x \)  
   b. \( f(x) = \ln(x + 1) \)  
   c. \( f(x) = \ln(3x - 1) \)

12
For the given functions \( f(x) \), let \( x_0 = 1, x_1 = 1.25 \), and \( x_2 = 1.6 \). Construct interpolation polynomials
of degree at most one and at most two to approximate \( f(1.4) \), and find the absolute error.

a. \( f(x) = \sin \pi x \)  
   b. \( f(x) = \log_{10}(3x - 1) \)  
   c. \( f(x) = \log_{10}(3x - 1) \)

13
Let \( P_1(x) \) be the interpolating polynomial for the data \((0,0), (0.5, y), (1, 3)\), and \((2, 2)\). The coefficient
of \( x^2 \) in \( P_1(x) \) is 6. Find \( y \).

14
Neville’s method is used to approximate \( f(0.4) \), giving the following table.

| \( x_0 = 0 \) | \( P_0 = 1 \) |
| \( x_1 = 0.25 \) | \( P_1 = 2 \) |
| \( x_2 = 0.5 \) | \( P_2 = 2.6 \) |
| \( x_3 = 0.75 \) | \( P_3 = 2.4 \) |
| \( x_4 = 1 \) | \( P_4 = 2.96 \) |

Determine \( P_2 = f(0.5) \).

15
Neville’s method is used to approximate \( f(0.5) \), giving the following table.

| \( x_0 = 0 \) | \( P_0 = 0 \) |
| \( x_1 = 0.4 \) | \( P_1 = 2.8 \) |
| \( x_2 = 0.7 \) | \( P_2 = 3.5 \) |
| \( x_3 = 1 \) | \( P_3 = 27 \) |

Determine \( P_2 = f(0.7) \).

16
Neville’s Algorithm is used to approximate \( f(0) \) using \( f(-2), f(-1), f(1) \), and \( f(2) \). Suppose
\( f(-1) \) was understated by 2 and \( f(1) \) was understated by 3. Determine the error in the original
calculation of the value of the interpolating polynomial to approximate \( f(0) \).

17
Use the Newton forward-difference formula to construct interpolating polynomials of degree one,
two, and three for the following data. Approximate the specified value using each of the polynomials.

a. \( f(0.43) \) if \( f(0) = 1, f(0.2) = 1.64872, f(0.5) = 2.71828, f(0.75) = 4.48169 \)
   b. \( f(0.48) \) if \( f(0) = -0.29004986, f(0.2) = -0.56079734, f(0.3) = -0.81401972, f(0.4) = -1.0526302 \)

18
Use the Newton backward-difference formula to construct interpolating polynomials of degree one,
two, and three for the following data. Approximate the specified value using each of the polynomials.

a. \( f(0.43) \) if \( f(0) = 1, f(0.2) = 1.64872, f(0.5) = 2.71828, f(0.75) = 4.48169 \)
   b. \( f(0.25) \) if \( f(-1) = 0.86199480, f(-0.5) = 0.95802099, f(0) = 1.0986123, f(0.5) = 1.29433767 \)

19
Determine the natural cubic spline \( S \) that interpolates the data \( f(0) = 0, f(1) = 1, \) and \( f(2) = 2 \).

20
Determine the clamped cubic spline \( s \) that interpolates the data \( f(0) = 0, f(1) = 1, f(2) = 2 \) and
satisfies \( s(0) = s(2) - 1 \).
UNIT-III

21

Use the forward-difference formulas and backward-difference formulas to determine each missing entry in the following tables.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.4794</td>
<td></td>
<td>0.0</td>
<td>0.00000</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.5646</td>
<td></td>
<td>0.2</td>
<td>0.74140</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>0.6442</td>
<td></td>
<td>0.4</td>
<td>1.3718</td>
<td></td>
</tr>
</tbody>
</table>

22

Derive a method for approximating \( f''(x_0) \) whose error term is of order \( h^4 \) by expanding the function \( f \) in a fourth Taylor polynomial about \( x_0 \) and evaluating at \( x_0 \pm h \) and \( x_0 \pm 2h \).

23

The forward-difference formula can be expressed as

\[
f'(x_0) = \frac{1}{h} [f(x_0 + h) - f(x_0)] - \frac{h}{2} f''(x_0) + \frac{h^2}{6} f'''(x_0) + O(h^3),
\]

Use extrapolation to derive an \( O(h^3) \) formula for \( f'(x_0) \).

24

Show that

\[
\lim_{h \to 0} \left( \frac{2 + h}{2 - h} \right)^{1/h} = e.
\]

25

Approximate the following integrals using the Trapezoidal rule.

a. \[
\int_{0.5}^{1} x^4 \, dx
\]

b. \[
\int_{0}^{0.5} \frac{2}{x-3} \, dx
\]

c. \[
\int_{1}^{1.5} x^2 \ln x \, dx
\]

d. \[
\int_{0}^{1} x^2 e^{-x} \, dx
\]

26

The Trapezoidal rule applied to \( \int_{0}^{1} f(x) \, dx \) gives the value 5, and the Midpoint rule gives the value 4. What value does Simpson’s rule give?

27

The quadrature formula \( \int_{0}^{1} f(x) \, dx = c_0 f(0) + c_1 f(1) + c_2 f(2) \) is exact for all polynomials of degree less than or equal to 2. Determine \( c_0, c_1, \) and \( c_2 \).

28

Romberg integration is used to approximate

\[
\int_{2}^{3} f(x) \, dx.
\]

If \( f(2) = 0.51342, f(3) = 0.36788, R_{11} = 0.43687, \) and \( R_{33} = 0.43626 \), find \( f(2.5) \).

29

Use Romberg integration to compute \( R_{11} \) for the following integrals.

a. \[
\int_{1}^{1.4} x^2 \ln x \, dx
\]

b. \[
\int_{0}^{1} x^2 e^{-x} \, dx
\]

30

Use Romberg integration to compute \( R_{11} \) for the following integrals.

a. \[
\int_{1}^{1.4} \cos x^2 \, dx
\]

b. \[
\int_{0}^{1} x \ln(x + 1) \, dx
\]
COMPLEX ANALYSIS

Theory: 3 credits and Practicals: 1 credit
Theory: 3 hours /week and Practicals: 2 hours /week

Objective: Analytic Functions, contour integration and calculus of residues will be introduced to the students.
Outcome: Students realize calculus of residues is one of the power tools in solving some problems, like improper and definite integrals, effortlessly.

Unit -- I

Regions in the Complex Plane - Analytic Functions - Functions of a Complex Variable - Mappings - Mappings by the Exponential Function - Limits - Theorems on Limits - Limits Involving the Point at Infinity - Continuity - Derivatives - Differentiation Formulas - Cauchy–Riemann Equations - Sufficient Conditions for Differentiability - Polar Coordinates-Harmonic Functions

Elementary Functions: The Exponential Function - The Logarithmic Function - Branches and Derivatives of Logarithms - Some Identities Involving Logarithms - Complex Exponents - Trigonometric Functions - Hyperbolic Functions

Unit -- II

Integrals: Derivatives of Functions w(t) - Definite Integrals of Functions w(t) - Contours - Contour Integrals - Some Examples - Examples with Branch Cuts - Upper Bounds for Moduli of Contour Integrals - Antiderivatives

Unit -- III

Text: James Ward Brown and Ruel V. Churchill, *Complex Variables and Applications* (8e)

References: Joseph Bak and Donald J Newman, *Complex analysis*

Lars V Ahlfors, *Complex Analysis*

S. Lang, *Complex Analysis*

B Choudary, *The Elements Complex Analysis*
UNIT-1

Practicals Question Bank

1. Sketch the following sets and determine which are domains:
   (a) \(|z - 2 + i| \leq 1\);
   (b) \(|2z + 3| > 4\);
   (c) \(\text{Im} \ z > 1\);
   (d) \(\text{Im} \ z = 1\);

2. Sketch the region onto which the sector \(r \leq 1, 0 \leq \theta \leq \pi/4\) is mapped by the transformation (a) \(w = z^2\); (b) \(w = z^3\); (c) \(w = z^4\).

3. Find all roots of the equation:
   (a) \(\sinh z = 1\);
   (b) \(\cosh z = \frac{1}{2}\).

4. Find all values of \(z\) such that
   (a) \(e^z = -2\);
   (b) \(e^z = 1 + \sqrt{3}i\);
   (c) \(\exp(2z - 1) = 1\).

5. Show that
   \[\lim_{z \to z_0} f(z)g(z) = 0\] if \(\lim_{z \to z_0} f(z) = 0\).
   and if there exists a positive number \(M\) such that \(|g(z)| \leq M\) for all \(z\) in some neighborhood of \(z_0\).

6. Show that \(f'(z)\) does not exist at any point if
   (a) \(f(z) = z\);
   (b) \(f(z) = z - \bar{z}\);
   (c) \(f(z) = 2x + ixy^2\);
   (d) \(f(z) = e^z e^{-\bar{z}}\).

7. Verify that each of these functions is entire:
   (a) \(f(z) = 3x + iy + i3y - x\);
   (b) \(f(z) = \sinh x \cosh y + i \cos x \sinh y\);
   (c) \(f(z) = e^{-x} \sinh x - i e^{-y} \cos y\);
   (d) \(f(z) = (z^2 - 2i) e^{-\bar{z}}\).

8. State why a composition of two entire functions is entire. Also, state why any linear combination \(c_1 f_1(z) + c_2 f_2(z)\) of two entire functions, where \(c_1\) and \(c_2\) are complex constants, is entire.

9. Show that \(u(x, y)\) is harmonic in some domain and find a harmonic conjugate \(v(x, y)\) when
   (a) \(u(x, y) = 2x(1 - y)\);
   (b) \(u(x, y) = 2x - y^4 + 3xy^2\);
   (c) \(u(x, y) = \sinh x \sin y\);
   (d) \(u(x, y) = y / (x^2 + y^2)\).

10. Show that if \(u\) and \(V\) are harmonic conjugates of \(u(x, y)\) in a domain \(D\), then \(u(x, y)\) and \(V(x, y)\) can differ at most by an additive constant.
evaluate
\[ \int_C f(z) \, dz. \]

\( f(z) = (z + 2)/z \) and \( C \) is
(a) the semicircle \( z = 2e^{it} \ (0 \leq \theta \leq \pi) \);
(b) the semicircle \( z = 2e^{it} \ (\pi \leq \theta \leq 2\pi) \);
(c) the circle \( z = 2e^{it} \ (0 \leq \theta \leq 2\pi) \).

12

\( f(z) \) is defined by means of the equations
\[ f(z) = \begin{cases} 1 & \text{when } y < 0, \\ 4 & \text{when } y > 0. \end{cases} \]

and \( C \) is the arc from \( z = -1 - i \) to \( z = 1 + i \) along the curve \( y = x^3 \).

13
Let \( C \) denote the line segment from \( z = i \) to \( z = 1 \). By observing that of all the points on that line segment, the midpoint is the closest to the origin, show that
\[ \left| \int_C \frac{dz}{z^2} \right| \leq 4\sqrt{2} \]
without evaluating the integral.

14
Let \( C \) denote the upper half of the circle \( |z| = R \ (R > 2) \), taken in the counterclockwise direction. Show that
\[ \left| \int_{C_R} \frac{2z^2 - 1}{2z + 5z^2 + 4} \, dz \right| \leq \frac{\pi R(2R^2 + 1)}{(R^2 - 1)(R^2 - 4)}. \]

Then, by dividing the numerator and denominator on the right here by \( R^4 \), show that the value of the integral tends to zero as \( R \) tends to infinity.

15
By finding an antiderivative, evaluate each of these integrals, where the path is any contour between the indicated limits of integration:
(a) \( \int_{\Gamma} e^z \, dz \); \( \int_{\Gamma} \cos \left( \frac{z}{2} \right) \, dz \); \( \int_{\Gamma} (z - 2)^4 \, dz \).

16
Use an antiderivative to show that for every contour \( C \) extending from a point \( z_1 \) to a point \( z_2 \),
\[ \int_C e^z \, dz = \frac{1}{n+1} (z_2^{n+1} - z_1^{n+1}) \quad (n = 0, 1, 2, \ldots). \]
Let $C_t$ and $C$ denote the circles

$$z = z_0 + Re^{i\theta} \quad (-\pi \leq \theta \leq \pi) \quad \text{and} \quad z = Re^{i\theta} \quad (-\pi \leq \theta \leq \pi),$$

respectively.

(a) Use these parametric representations to show that

$$\int_{C_t} f(z - z_0) \, dz = \int_{C} f(z) \, dz$$

18

evaluate the integral

$$\int_{C} z^m z^n \, dz,$$

where $m$ and $n$ are integers and $C$ is the unit circle $|z| = 1$, taken counterclockwise.

19

$f(z) = 1$ and $C$ is an arbitrary contour from any fixed point $z_1$ to any fixed point $z_2$ in the $z$ plane.

evaluate

$$\int_{C} f(z) \, dz.$$

20

$f(z) = \pi \exp(\pi z)$ and $C$ is the boundary of the square with vertices at the points $0$, $1$, $1 + i$, and $i$, the orientation of $C$ being in the counterclockwise direction.

evaluate

$$\int_{C} f(z) \, dz.$$

UNIT-III

21

Let $C$ denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of these integrals:

(a) $\int_{C} \frac{e^z}{z - (\pi i/2)} \, dz$ ;
(b) $\int_{C} \frac{\cos z}{z(z^2 + 8)} \, dz$ ;
(c) $\int_{C} \frac{z \, dz}{2z + 1}$

22

Find the value of the integral of $g(z)$ around the circle $|z - i| = 2$ in the positive sense when

(a) $g(z) = \frac{1}{z^2 + 4}$;
(b) $g(z) = \frac{1}{(z^2 + 4)^2}$.

23

Let $C$ be the circle $|z| = 3$, described in the positive sense. Show that if

$$g(z) = \int_{C} \frac{2\sqrt{\gamma - z}}{3 - z} \, dz \quad (|z| \neq 3),$$

then $g(2i) = 8\pi i$. What is the value of $g(z)$ when $|z| = 3$?

24

Let $C$ be any simple closed contour, described in the positive sense in the $z$ plane, and write

$$v(r) = \int_{C} \frac{\sqrt{r^2 + 2r}}{r^3} \, dz.$$ 

Show that $g(z) = 6\pi i$ when $z$ is inside $C$ and that $v(z) = 0$ when $z$ is outside.
25
Show that if \( f \) is analytic within and on a simple closed contour \( C \) and \( z_0 \) is not on \( C \), then
\[
\int_C \frac{f'(z)}{z - z_0} \, dz = \int_C \frac{f(z)}{(z - z_0)^2} \, dz.
\]

26
Let \( C \) be the unit circle \( z = e^{it} (-\pi \leq t \leq \pi) \). First show that for any real constant \( a \),
\[
\int_C \frac{e^{at}}{z} \, dz = 2\pi i
\]
Then write this integral in terms of \( \theta \) to derive the integration formula
\[
\int_0^\pi e^{a\cos\theta} \cos(a \sin \theta) \, d\theta = \pi.
\]

27
Suppose that \( f(z) \) is entire and that the harmonic function \( u(x, y) = \text{Re} |f(z)| \) has an upper bound \( u_0 \); that is, \( u(x, y) \leq u_0 \) for all points \((x, y)\) in the \( xy \) plane. Show that \( u(x, y) \) must be constant throughout the plane.

28
Let a function \( f \) be continuous on a closed bounded region \( R \), and let it be analytic and not constant throughout the interior of \( R \). Assuming that \( f(z) \neq 0 \) anywhere in \( R \), prove that \( |f(z)| \) has a minimum value \( m \) in \( R \) which occurs on the boundary of \( R \) and never in the interior. Do this by applying the corresponding result for maximum.

29
Let the function \( f(z) = u(x, y) + iv(x, y) \) be continuous on a closed bounded region \( R \), and suppose that it is analytic and not constant in the interior of \( R \), Show that the component function \( u(x, y) \) has maximum and minimum values in \( R \) which are reached on the boundary of \( R \) and never in the interior, where it is harmonic.

30
Let \( f \) be the function \( f(z) = e^z \) and \( R \) the rectangular region \( 0 \leq x \leq 1, 0 \leq y \leq \pi \). Illustrate results in Sec. 54 and Exercise 6 by finding points in \( R \) where the component function \( u(x, y) = \text{Re} |f(z)| \) reaches its maximum and minimum values.
25
Show that if $f$ is analytic within and on a simple closed contour $C$ and $z_0$ is not on $C$, then
\[ \int_C \frac{f'(z)}{z - z_0} \, dz = \int_C \frac{f(z)}{(z - z_0)^2} \, dz. \]

26
Let $C$ be the unit circle $z = e^{it} (-\pi \leq t \leq \pi)$. First show that for any real constant $a$,
\[ \int_C \frac{e^{zt}}{z} \, dz = 2\pi i. \]
Then write this integral in terms of $\theta$ to derive the integration formula
\[ \int_0^\pi e^{i\cos \theta} \cos (a \sin \theta) \, d\theta = \pi. \]

27
Suppose that $f(z)$ is entire and that the harmonic function $u(x, y) = \text{Re} \left| f(z) \right|$ has an upper bound $u_0$; that is, $u(x, y) \leq u_0$ for all points $(x, y)$ in the $xy$ plane. Show that $u(x, y)$ must be constant throughout the plane.

28
Let a function $f$ be continuous on a closed bounded region $R$, and let it be analytic and not constant throughout the interior of $R$. Assuming that $f(z) \neq 0$ anywhere in $R$, prove that $|f(z)|$ has a minimum value $m$ in $R$ which occurs on the boundary of $R$ and never in the interior. Do this by applying the corresponding result for maximum.

29
Let the function $f(z) = u(x, y) + iv(x, y)$ be continuous on a closed bounded region $R$, and suppose that it is analytic and not constant in the interior of $R$. Show that the component function $u(x, y)$ has maximum and minimum values in $R$ which are reached on the boundary of $R$ and never in the interior, where it is harmonic.

30
Let $f$ be the function $f(z) = e^z$ and $R$ the rectangular region $0 \leq x \leq 1, 0 \leq y \leq \pi$. Illustrate results in Sec. 54 and Exercise 6 by finding points in $R$ where the component function $u(x, y) = \text{Re} \left| f(z) \right|$ reaches its maximum and minimum values.
UNIT-1
1 Evaluate the line integral
\[ \int_C \mathbf{F} \times d\mathbf{r}, \]
where \( \mathbf{F} \) is the vector field \((y, x, 0)\) and \( C \) is the curve \( y = \sin x, \; z = 0, \) between \( x = 0 \) and \( x = \pi. \)

2 Evaluate the line integral
\[ \int_C x + y^2 \, d\mathbf{r}, \]
where \( C \) is the parabola \( y = x^2 \) in the plane \( z = 0 \) connecting the points \((0, 0, 0)\) and \((1, 1, 0)\).

3 Evaluate the line integral
\[ \int_C \mathbf{F} \cdot d\mathbf{r} \quad \text{where} \quad \mathbf{F} = (5x^2, 2x, x + 2y) \]
and the curve \( C \) is given by \( x = t, \; y = t^2, \; z = t^3, \; 0 \leq t \leq 1. \)

4 Find the line integral of the vector field \( \mathbf{u} = (y^2, x, z) \) along the curve given by \( z = y = e^t \) from \( x = 0 \) to \( x = 1. \)

5 Evaluate the surface integral of \( \mathbf{u} = (y, x^2, z^2) \), over the surface \( S \), where \( S \) is the triangular surface on \( z = 0 \) with \( y \geq 0, \; z \geq 0, \; y + z \leq 1, \) with the normal \( \mathbf{n} \) directed in the positive \( x \) direction.

6 Find the surface integral of \( \mathbf{u} = \mathbf{r} \) over the part of the paraboloid \( z = 1 - x^2 - y^2 \) with \( z > 0, \) with the normal pointing upwards.

7 If \( S \) is the entire \( x, y \) plane, evaluate the integral
\[ I = \iint_S e^{-x^2 - y^2} \, dS, \]
by transforming the integral into polar coordinates.

8 Find the line integral \( \oint_C \mathbf{F} \times d\mathbf{r} \) where the curve \( C \) is the ellipse \( x^2/a^2 + y^2/b^2 = 1 \) taken in an anticlockwise direction. What do you notice about the magnitude of the answer?

9 By considering the line integral of \( \mathbf{F} = (y, x^2 - x, 0) \) around the square in the \( x, y \) plane connecting the four points \((0, 0), (1, 0), (1, 1) \) and \((0, 1)\), show that \( \mathbf{F} \) cannot be a conservative vector field.

10 Evaluate the line integral of the vector field \( \mathbf{u} = (xy, z^2, x) \) along the curve given by \( x = 1 + t, \; y = 0, \; z = t^4, \; 0 \leq t \leq 3 \).
UNIT II

11. A cube $0 \leq x, y, z \leq 1$ has a variable density given by $\rho = 1 + x + y + z$. What is the total mass of the cube?

12. Find the volume of the tetrahedron with vertices at $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$ and $(a, b, c)$.

13. Evaluate the surface integral of $u = (xy, z, x + y)$ over the surface $S$ defined by $z = 0$ with $0 \leq x \leq 1$, $0 \leq y \leq 2$, with the normal $\mathbf{n}$ directed in the positive $z$ direction.

14. Find the surface integral of $u = r$ over the surface of the unit cube $0 \leq x, y, z \leq 1$, with $\mathbf{n}$ pointing outward.

15. The surface $S$ is defined to be that part of the plane $z = 0$ lying between the curves $y = x^2$ and $x = y^2$. Find the surface integral of $u \cdot \mathbf{n}$ over $S$ where $u = (x, xy, x^2)$ and $\mathbf{n} = (0, 0, 1)$.

16. Find the surface integral of $u \cdot \mathbf{n}$ over $S$ where $S$ is the part of the surface $z = x + y^2$ with $z < 0$ and $x > -1$, $u$ is the vector field $u = (2y + z, -1, 0)$ and $\mathbf{n}$ has a negative $z$ component.

17. Find the volume integral of the scalar field $\phi = x^2 + y^2 + z^2$ over the region $V$ specified by $0 \leq x \leq 1$, $1 \leq y \leq 2$, $0 \leq z \leq 3$.

18. Find the volume of the section of the cylinder $x^2 + y^2 = 1$ that lies between the planes $z = x + 1$ and $z = -x - 1$.

19. Find the unit normal $\mathbf{n}$ to the surface $x^2 + y^2 - z = 0$ at the point $(1, 1, 2)$.

20. Find the gradient of the scalar field $f = xyz$, and evaluate it at the point $(1, 2, 3)$. Hence find the directional derivative of $f$ at this point in the direction of the vector $(1, 1, 0)$.

UNIT III

21. Find the divergence of the vector field $u = r$.

22. The vector field $u$ is defined by $u = (xy, z + x, y)$. Calculate $\nabla \times u$ and find the points where $\nabla \times u = 0$.

23. Find the gradient $\nabla \phi$ and the Laplacian $\nabla^2 \phi$ for the scalar field $\phi = x^2 + xy + yz^2$.

24. Find the gradient and Laplacian of $\phi = \sin(kx) \sin(ty) \exp(\sqrt{k^2 + t^2}z)$.

25. Find the unit normal to the surface $xy^2 + 2yz = 4$ at the point $(-2, 2, 3)$. 

355
26
For \( \phi(x, y, z) = x^2 + y^2 + z^2 + xy - 3x \), find \( \nabla \phi \) and find the minimum value of \( \phi \).

27
Find the equation of the plane which is tangent to the surface \( x^2 + y^2 - 2z^3 = 0 \) at the point \((1, 1, 1)\).

28
Find both the divergence and the curl of the vector fields
(a) \( u = (y, z, x) \);
(b) \( v = (xyz, z^2, x - y) \).

29
For what values, if any, of the constants \( a \) and \( b \) is the vector field
\( u = (y \cos x + axz, b \sin x + z, x^2 + y) \) irrotational?

30
(a) Show that \( u = (y^2 z, -z^2 \sin y + 2xyz, 2x \cos y + y^2 x) \) is irrotational.
(b) Find the corresponding potential function.
(c) Hence find the value of the line integral of \( u \) along the curve
\( x = \sin \pi t/2, y = t^2 - t, z = t^4, 0 \leq t \leq 1 \).
A set of MOOCs resources for ICT based learning and teaching

NPTEL: nptel.ac.in

COURSERA: www.coursera.org

MITOCW: ocw.mit.edu

ACADEMIC EARTH: www.academicearth.org

EdX: www.edx.org

KHAN ACADEMY: www.khanacademy.org

ALISON: www.alison.com

STANFORD ONLINE: www.online.stanford.edu

VIDEO LECTURES: videolectures.net

INTERACTIVE REAL ANALYSIS: mathcs.org

VISUAL CALCULUS: archives.math.utk.edu/visual.calculus

MOOCS CALCULUS: mooculus.osu.edu

Few Math Softwares

Useful for Classroom teaching: Geogebra (Freeware)

Type setting software: LaTeX

High end commercial softwares: Mathematica, Maple, Matlab

Answering search engine: www.wolframalpha.com

Group theory software: group explorer 2.2 (Freeware)

Visualization software: Mathematics Visualization Toolkit (freeware)